Short Term Debt and Bank Liability Structure☆ (preliminary, comments welcome)

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Abstract

We build a dynamic model with the aim of providing insight into the determinants of a bank's liability structure. In our model, bank assets consist of risky illiquid loans and liquid reserves and they are financed by a combination of (i) insured deposits, (ii) short–term, secured debt that is subject to rollover risk, (iii) long–term, risky debt and (iv) equity. Our analysis shows that the rollover risk implied by short–term funding adds to the cost of long–term debt, which prevents the bank from an aggressive use of short-term debt. In contrast, poor returns on risky assets, abundant deposit funding and the depositor preference rule do exacerbate the bank's appetite for cheap but unstable short-term debt. In addition, we study the impact of three regulatory tools, namely liquidity coverage ratio, payout restrictions and leverage ratio, and show that each of these tools is able to curb the bank's reliance on short–term debt.

Keywords: Banks; regulation; rollover risk; short–term debt.
JEL: G21; G28; G32; G33.

1. Introduction

What drives the financing decisions of modern banks? The academic literature had, until recently, paid little attention to banks' financing decisions. However, recent empirical evidence suggests that banks have a far richer liability structure than a simple mix of equity and insured deposits.[1] In particular, in the years preceding the Global Financial Crisis, short–term wholesale debt gained increasing popularity, since it was considered a relatively cheap source of funding.[2] This cost advantage was particularly relevant in the case of repos due to their deposit–like nature, safe harbor provisions and the investors' preferences for safe and liquid investment. However, the example of Northern Rock, brought to the edge of failure in 2008 by a run of wholesale creditors, clearly shows that overreliance on short–term funding may render a bank particularly fragile.

In this paper we develop a theoretical framework to elucidate the main drivers of bank financing and, in particular, debt–maturity structures. In our model, a bank can be financed by a combination of (i) insured

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[1] According to Gropp and Heider (2010), non–deposit sources of funding constituted about 30% of the liabilities of European banks in 2004. The snapshot of the U.S. bank holding companies reported in Hanson et al. (2011) suggests that deposits amount to roughly half of a bank's liabilities, whereas the remainder represents a mix of wholesale/retail non–deposit funding and equity.

[2] For example, Chernenko and Sunderam (2014) provide explicit evidence of the growing importance of short–term funding for European banks: in 2011, 18% of total assets of U.S. money market funds were short–term loans attributed to European banks.

deposits, (ii) short–term, secured debt, (iii) long–term, risky debt and (iv) equity. The main question we address deals with the choices between short–term, secured debt and long–term, risky debt. The former is relatively cheap but is subject to an exogenous run of short–term creditors, whereas the latter represents a stable but costlier source of funding. The asset side of the bank's balance sheet consists of risky, illiquid assets and liquid reserves. The level of liquid reserves fluctuates over time and is controlled via a payout policy. The bank may draw from its liquid reserves to ensure the continuity of interest payments on debt. Moreover, liquid reserves serve as a buffer against the withdrawal losses caused by the run of short-term creditors. These two features lead to an interaction between the bank's financing structure and its payout policy, which turns out to be crucial for our results. The bank's default policy is also related to the dynamics of its liquid reserves: in the unregulated environment, running out of liquid reserves triggers the bank's default. On the other hand, when the bank faces capital or liquidity requirements, the default might occur at strictly positive levels of liquid reserves.

We begin by providing a formal characterization of the bank's optimal financing and payout decisions, which in our framework are jointly determined. As it is the case in many inventory models (see e.g., Jeanblanc and Shiryaev (1995) and Décamps et al. (2011)) and in recent contributions that simultaneously deal with financing and payout decisions (see e.g., Hugonnier and Morellec (2014) and Bolton et al. (2014)), the bank retains earnings below a certain level of liquid reserves and only distributes dividends whenever its liquid reserves reach this target level. The choice of the latter depends on the bank's financing structure, which affects the cost of financing and, thus, the dynamics of bank earnings. On the other hand, the choice of the financing structure itself depends on the target level of liquid reserves, since the latter determines the cost of long–term risky debt.

We first solve the model numerically for a hypothetical bank that has no deposits in its financing structure and faces no regulation. Our numerical analysis shows that the presence of short–term, secured debt in the bank's financing structure entails an additional component in the spread of the long–term debt, in the sequel labeled *shadow costs*. These shadow costs may be interpreted as a kind of premium rewarding long–term creditors for the negative externalities imposed on them by the (possible) run of short–term debt creditors. The presence of the shadow costs curbs the bank's incentives to excessively rely on cheap but unstable short–term debt. Interestingly, the magnitude of the shadow costs is closely related to the bank's capacity to maintain high levels of liquid reserves. In particular, banks with higher returns on risky assets face lower shadow costs, as they are able, by increasing their target level of liquid reserves, to more effectively mitigate the additional default risk related to the use of short–term debt. For these banks, the relative cost advantage of short–term debt is not substantial and, therefore, they exhibit a lower proportion of it in their financing structure. In contrast, banks with lower returns on risky asset cannot afford to implement substantial upward adjustments of liquid reserves and, as a consequence, face higher shadow costs, which magnifies the initial cost advantage of short–term, secured debt over long–term, risky debt. As a result, banks with lower returns on risky assets exhibit higher proportions of short–term debt in their financing structure.

On the second stage of our numerical analysis, we reexamine the bank's financing decisions allowing for access to insured deposit funding, whose volume is taken as given. Notably, the access to abundant deposit funding induces the bank to increase its reliance on short–term debt. The explanation for this effect is the following: A larger volume of insured deposits in the bank's liability structure reduces the effective bank earnings, hence, it weakens the bank's capacity to maintain high levels of liquid reserves. This amplifies the costs of long–term debt financing and induces the bank to increase its reliance on short–term debt. Moreover, this effect becomes even more pronounced when insured deposits are senior to long–term, risky debt. The seniority of insured deposit over long–term debt implies that long–term creditors receive a lower value in the case of bank liquidation and, thus, demand a higher interest rate, which amplifies the cost advantage of short–term debt financing even further.

After studying the financing decisions of the unregulated bank, we consider the impact of different regulatory measures on the ex–ante choice of the bank's financing structure. After the Global Financial Crisis, over–reliance on short–term sources of funding in the banking sector is considered to be socially dangerous and thus represents a serious concern for bank regulators. In this light, it might be useful to gain a better understanding of how different regulatory measures affect banks' appetite for short–term funding.

We examine the effects of three regulatory tools: liquidity regulation in the spirit of the Basel III Liquidity Coverage Ratio, payout restrictions and leverage ratio. Overall, our analysis shows that each of these tools is capable of curbing the banks' reliance on short–term debt. Under liquidity regulation, which requires the bank to maintain a minimum level of liquid reserves as a certain proportion of its volume of short–term debt, the bank substitutes short–term debt funding by long–term debt. However, such a substitution results in no increase of the bank's default probability. Payout restrictions induce a similar substitution effect: under payout restrictions, the bank operates with higher target levels of liquid reserves, which reduces the cost of long–term debt financing and, thus, the cost advantage of short–term debt. Leverage regulation, however, induces the bank to lower the volumes of both short–term and long–term debt. Put into the perspective of policy debates, this set of results suggests that developing special liquidity regulation in order to curb banks' reliance on short–term debt might be redundant.

Our paper belongs to the burgeoning body of literature that examines the interaction between optimal capital structure and liquidity management (see e.g., Bolton et al. (2014), Gryglewicz (2011) and Hugonnier and Morellec (2014)). In these works, capital structure affects liquidity reserves through debt interest payments. However, these models allow for a single type of debt (perpetual debt) in the financing structure, thereby disregarding the dimension of debt–maturity choice. In our model, we introduce an additional type of debt with shorter maturity and bring into focus the trade–off between the costs of funding and stability. Allowing for short–term debt in the bank's financing structure also creates an additional channel of interaction between liquidity management and capital structures, given that liquid reserves serve as a buffer against the withdrawal losses generated by a run of the short–term creditors. This feature plays an important role in our analysis, since the ability of the bank to build larger liquid reserves has direct implications on its optimal debt–structure choice. At the same time, it marks a key difference between our framework and the existing structural models that deal with rollover risk such as He and Xiong (2012) and He and Milbradt (2014a, b), where rollover losses are absorbed by the shareholders' "deep pockets". These models, however, focus on the impact of rollover risk on the endogenous default decisions. In our framework, default occurs as soon as the liquid reserves are depleted, but the probability of running out of liquidity is strongly affected by both the payout and financing decisions of the bank.

An additional key departure of our work from the papers by He and Xiong (2012), He and Milbradt (2014a) and He and Milbradt (2014b) pertains to the approach to modeling rollover risk. In their models, all the debt is risky and the maturities of bonds are uniformly distributed (as in the setting of Leland and Toft (1996)). Maturing debt has to be replaced at a market price, which might be above or below its face value, thereby leading to rollover gains/losses. In the present paper we adopt an alternative approach to modeling rollover risk by considering its materialization as an *exogenous* run of short–term creditors. This is motivated by the anecdotal evidence that the large actors of short–term debt markets, such as money–market mutual funds, and even traditional banks, may face sudden liquidity needs caused by a run of their own clients, which, in turn, will push them to pull their short–term investments back.

Our paper is also related to the literature on optimal maturity composition of debt. While exploring this issue, most papers place emphasis on the role that short–term debt plays in resolving various kinds of agency problems (see e.g., Eisenbach (2013), Chen et al. (2014), Diamond and He (2014)). We assume, instead, that the only benefit of using short–term secured debt stems from its lower cost and focus on the impact that its presence has on the costs of long–term debt. In this respect, our model is complementary to the work by Auh and Sundaresan (2013), who study a similar question in the context of a Leland–type structural model. The main feature that distinguishes our model from theirs is the fact that we allow for the joint choice of the optimal debt structure and liquidity management and, thereby, capture the link between the composition of the asset and liability sides of the balance sheet. In a recent contribution, He and Milbradt (2014a) study the fully dynamic choice of optimal debt maturity in a setting à la Leland and Toft (1996). In their model, a firm can modify its average debt maturity over time by substituting a fraction of the maturing long-term bonds by short–term ones (or vice versa). Increasing the fraction of debt with shorter maturity reduces current rollover losses, as it has a lower default risk and, thus, a higher price than debt with a longer maturity. However, this also increases the rollover frequency in the future, thereby, increasing the default probability. A very similar trade–off between the lower cost of funding and the additional default risk inherent to short-term funding is also a key driver of the results in our setting. Nevertheless, the fact that our debt maturity structure is static enables us to address the optimal choice of financing structure, whereas in He and Milbradt (2014b) the latter is taken as exogenous.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 provides a formal characterization of the bank policies. In Section 4 we conduct a numerical analysis to identify the determinants of bank financing structure. In Section 5 we examine the impact of different regulatory measures on the bank funding decisions. Section 6 concludes. All mathematical proofs can be found in the Appendix.

2. The Model

We work in a continuous–time, infinite–horizon setting, in which all agents are risk neutral and the risk–free rate is $\rho > 0$. A group of equity investors holds a banking license and has to decide on the financing structure of a new bank. The bank's assets comprise *liquid reserves* and a portfolio of *risky assets* of a fixed size A. A fraction $(1-\eta)A$ of the risky assets, for $\eta \in (0,1)$ given, consists of illiquid loans that have zero value in the case of bank liquidation[3]. The remaining fraction ηA corresponds to assets that hold their value in the case of bank liquidation and, thus, can be used as collateral for secured borrowing transactions (e.g., sovereign bonds, agency CMOs, some approved ABS, etc.). Once the bank is established, the risky assets generate the after–tax cash–flows

$$(1-\theta)(\mu dt + \sigma dW(t)),$$

where $\theta \in (0,1)$ is the tax rate, μ denotes the expected return on risky assets, σ reflects the volatility of asset returns and $W = \{W(t), t \geq 0\}$ is a standard Brownian motion defined on the standard probability space $(\Omega, \mathcal{F}, \mathrm{P})$ that generates the filtration $\mathbb{F} = \{\mathcal{F}_t, t \geq 0\}$.

The financing structure. The bank's assets can be financed using a mixture of insured deposits and the following types of securities: repos, long–term, risky debt and equity. S as to introduce the risk of liquidation in our model, we assume that there is no access to capital markets after the bank has been created[4]. When choosing the bank's financing structure, the shareholders take the cost and the volume of insured deposits as given and decide on how to complement the latter with non–deposit sources of funding. Throughout the paper, we focus our analysis on the choice of the initial financing structure, without allowing for further adjustments. Although this rigid structure is assumed for technical reasons, it can be justified using the empirical evidence that suggests that the capital structure of a bank is highly persistent and changes slowly over time (see e.g., IMF Global Financial Stability Report (2013)).

In order to formally state the decision problem of the bank's shareholders at the time of investment, we start by describing the menu of funding sources and their respective costs. Next, we characterize the dynamics of liquid reserves and compute the values of the bank's securities.

The types of debt funding and their corresponding costs. Deposits in our model are insured by a deposit–insurance fund and take the form of perpetual debt with a face value of P_d. The overall cost of deposit funding to the bank amounts to $r_d P_d dt$ per unit of time, where the rate $r_d < \rho$ encompasses the rate of the interest payments made to depositors as well as the deposit–insurance and management costs. Throughout the paper both P_d and r_d are taken as exogenous.

The non–deposit sources of financing consist of repos with a volume of $P_s \leq \eta A$ and long–term, risky debt with a face value of P_l. The repos are instantaneously rolled over, whereas the long–term debt takes the form of a perpetual bond. The repo creditors benefit from a so–called *safe–harbor provision*, which spares them from any losses in the case of bank liquidation (this is the case under the U.S. Bankruptcy Code). This

[3]This assumption stems from the fact that the value of bank loans largely depends on the bank's private information, which cannot be easily transmitted to a new owner. It can be relaxed with minor modifications.

[4]Allowing for uncertain recapitalization possibilities would not qualitatively alter our results.

implies that the long–term, risky debt is junior to the repos. However, the long–term debt may be junior or senior to the insured deposits. In the base case of our analysis, we consider the long–term, risky debt to be senior to the insured deposits. We then analyze the impact of the reverse seniority in Section 4.2.

The repo creditors are assumed to be subject to exogenous liquidity shocks, whose arrivals are described by the Poisson process $N = \{N(t), t \geq 0\}$ with intensity λ. When hit by a liquidity shock, the repo creditors stop rolling their debt over.[5] We define the (stopping) time when the repo creditors run as

$$\tau^* := \inf\{t > 0 : N(t) = 1\}.$$

For simplicity, we rule out the possibility of additional repo sales once the run of the incumbent creditors has occurred.[6]

The repo funding costs the bank $r_s P_s dt$ per unit of time. Since repos are fully collateralized, the interest rate $r_s < \rho$ does not reflect the bank's liquidation risk[7] and it is exogenous (e.g., a 1–year LIBOR). The long–term debt funding costs the bank $r_l P_l dt$ per unit of time, and the interest rate $r_l \geq \rho$ is chosen endogenously at the time of investment so as to compensate the long–term creditors for the liquidation risk they bear. It is worth mentioning that, even if the long–term debt were not subject to the liquidation risk, the long–term creditors would be compensated at a higher rate than the repo ones. The reason is that, in contrast to the repo creditors, the long–term ones are unable to withdraw their funds in the case of a liquidity shock. As a result, their default–free rate should comprise a kind of illiquidity premium. An alternative interpretation one may provide, based on standard liquidity arguments (see e.g., Diamond and Dybvig (1983), Gorton and Metrick (2010) and Admati and Hellwig (2013)), is that the repo creditors value the availability of funds and accept to be compensated at a lower rate.

As will become apparent below, the impact of repo funding on the overall cost of debt financing is ambiguous. On the one hand, repos may be viewed as a cheaper source of funding. On the other hand, increasing their proportion in the bank's liability structure simultaneously increases the risk of liquidation and reduces the value accruing to the unsecured creditors in such a case, thereby, amplifying the costs of long–term debt.

The dynamics of the liquid reserves. For a given cost structure of debt financing, the cumulative after–tax bank's earnings $R = \{R(t), t \geq 0\}$ evolve in the following way:

$$R(t) = (1-\theta)[(\mu - r_d P_d - r_l P_l - \mathbb{1}_{\{t \leq \tau^*\}} r_s P_s)t + \sigma W(t)],$$

where $\mathbb{1}_{\{.\}}$ is the zero–one indicator function reflecting the fact that, if it were to withstand a run of the repo creditors, the bank would continue operating without repos on its balance sheet. The presence of taxes implies that Modigliani–Miller does not hold; hence, the bank's value is not independent of its capital structure.

The after–tax earnings can be distributed to the bank's shareholders or they may be retained as liquid reserves. Maintaining liquid reserves, however, involves dead–weight costs that we capture via the assumption that no interest accrues on the retained earnings (this also allows for closed–form expressions for the values

[5]This situation may occur when the bank engages in repo transactions with a Money Market Mutual Fund (MMMF). As argued by Chernenko and Sunderam (2014), the MMMF investors' concerns about some of its assets may induce them to pull their money back, which triggers subsequent cuts in funding of totally credit–worthy firms financed through this fund (a negative spill–over effect). In other words, a materialization of the rollover risk may not necessarily be triggered by the investors' concerns about the quality of the bank' assets, but may simply represent a negative externality of the excessive risk–taking by the provider of short–term funding. This argument underlies the assumption of an exogenous run in our model.

[6]In fact, in the short run, the bank may be unable to replace its usual provider of short–term funding because of information asymmetries or even institutional frictions corresponding to the fact that the MMMF might be constrained by its board to lend only to a pre–approved number of counterparties (see Chernenko and Sunderam (2014)).

[7]We first study an unregulated setting, in which liquidation and default coincide. That is to say that the bank's shareholders will never default strategically at any positive level of liquidity, when the bank can still service its debt. When facing regulation, however, it might occur that the bank is subject to intervention before it becomes illiquid.

of the bank's liabilities). We model the cumulative payouts made to the shareholders up to time t via a $\mathbb{F}$–adapted, non–decreasing process $L = \{L(t), t \geq 0\}$. For any initial level of liquid reserves $c_0 \geq 0$, the financing and payout decisions can be described by a strategy $\pi = (P_s, P_l, L)$. We define by

$$\mathcal{A} := \{\pi \,:\, P_s \leq \eta A, \text{ and } L \text{ is non–decreasing}\}$$

the set of admissible strategies that can be adopted by the bank. The liquid–reserves process $C^\pi = \{C^\pi(t), t \geq 0\}$ associated to a strategy $\pi \in \mathcal{A}$ satisfies

$$C^\pi(t) = c_0 + R(t) - L(t) - \mathbb{1}_{\{t \leq \tau^*\}} P_s N(t), \tag{1}$$

where the last term on the right–hand side reflects the fact that the liquid reserves may be subject to a large negative shock whose scale corresponds to the volume of the withdrawn repo funding.[8]

We denote by $\underline{c} \geq 0$ the level of liquid reserves at which the bank is liquidated and by τ_π the corresponding liquidation time, defined for a given strategy π as

$$\tau_\pi := \inf\{t > 0 : C^\pi(t) < \underline{c}\}.$$

In the absence of regulation, and due to the limited liability of the bank's shareholders, it is suboptimal for them to liquidate the bank at a positive level of liquid reserves, hence $\underline{c} = 0$ (We provide a formal proof of this fact in AppendixA.5). However, as we show in Section 5, imposing liquidity or capital regulation on the bank may result in a strictly positive liquidation threshold, which would itself depend on the bank's financing structure.

It is important to notice that, in our setting, liquidation may be triggered by either the run of the repo creditors, should it occur when the level of liquid reserves $C^\pi(t) \in [\underline{c}, \underline{c} + P_s)$, or by a series of poor performances, in which case the liquid reserves get depleted gradually following adverse realizations of the Brownian risk. This shows that the role of the liquid reserves is twofold: First, they help the bank hedge against adverse profitability shocks and ensure the continuity of debt servicing. Second, they serve as a buffer against the large loss caused by the run of the repo creditors.

The shareholders' problem. In order to formalize the problem of the bank's shareholders, we first define the ex–ante value of equity corresponding to a strategy π and an initial level of liquid reserves c_0 :

$$U^\pi(c_0) := \mathbb{E}_0 \left[\int_0^{\tau_\pi} e^{-\rho t} dL(t) \right].$$

The term inside the conditional expectation reflects the present value of cumulated dividend payments to shareholders until the time of liquidation. In the absence of regulatory requirements, the shareholders walk away empty–handed in the case of liquidation. However, this is not necessarily the case in the presence of regulation, in which case the term $\mathbb{E}_0[\tilde{u}(C^\pi(\tau_\pi))]$ (the shareholders' payoff at liquidation) should be added to $U^\pi(c_0)$. We elaborate on this in Section 5.

The bank's shareholders choose among the set of all admissible financing and payout strategies $\mathcal{A}$, so as to maximize the value of equity, net of the initial investment expenditure. For a given choice of c_0, the total amount of funds that must be raised to establish the bank is $A + c_0$. The difference between the total investment costs and the funding raised, namely $(A + c_0) - (P_d + P_l + P_s)$, is financed by equity. The shareholders' optimization problem can then be stated as follows:

$$\max_{c_0 \geq 0, \pi \in \mathcal{A}} \quad V^\pi(c_0) := \Big\{ U^\pi(c_0) - (A + c_0 - P_d - P_l - P_s) \Big\}. \tag{2}$$

[8] We assume that, in the case of a run, all the repo creditors withdraw their funds simultaneously.

The ex–ante market value of the long–term debt under the strategy π is the expected value of the sum of the cumulated interest payments until the time of liquidation, plus the liquidation value accruing to the long–term creditors in such an event:

$$D^{\pi}(c_0) := \mathbb{E}_0\left[\int_0^{\tau_\pi} e^{-\rho t} r_l P_l dt + e^{-\rho\tau_\pi}\tilde{d}(C^{\pi}(\tau_\pi))\right].$$

The value $\tilde{d}(C^{\pi}(\tau_\pi))$ accruing to the long–term creditors in the event of liquidation depends on the seniority of the long–term debt relative to the insured deposits, as well as on whether or not the run of the repo creditors has already taken place. We assume that the long–term debt is issued in a competitive market with rational creditors, so that its interest rate is chosen so as to ensure the parity between its face and market values, i.e. $D^{\pi}(c_0) = P_l$, which implicitly defines the long–term interest rate r_l.

In order to provide some intuition regarding the solution to the shareholders' problem, we point out that the optimal payout strategy is of the so–called *barrier type*. As we formally show below, it is characterized by an optimal payout barrier (or, equivalently, an optimal target level of liquid reserves) such that all liquid reserves beyond the said barrier are distributed as dividends, while no payouts take place as long as the level of liquid reserves remains below this threshold. We will show that, in general,[9] the optimal initial level of liquid reserves c_0 coincides with the optimal target level of liquid reserves. This choice determines the ex–ante distance to liquidation and, therefore, impacts the cost of long–term debt financing via the relation $D^{\pi}(c_0) = P_l$. This is, in fact, the channel through which the payout policy affects the choice of the financing structure in our model. Simultaneously, one observes in the reserves–dynamics Equation (1) that the choice of financing structure feeds back into the dynamics of liquid reserves through the costs of debt servicing and, furthermore, through the scale of the rollover risk exposure.[10] As we show in the upcoming sections, this feedback mechanism plays an important role in determining the optimal financing structure of the bank.

3. The Financing and Payout Decisions of an Unregulated Bank

We consider, initially, the optimal payout policy and the values of the securities of a bank assuming it has survived the run of the repo creditors. The closed–form solutions for the values of equity and debt defined in this setting will be later used as building blocks for constructing the solution to the shareholders' problem at the time of the investment. Moreover, this setting will serve as a useful benchmark needed to assess the impact of the repo funding on the optimal financing and payout decisions of the bank's management in the numerical analysis that we perform in Section 4.

3.1. The payout policy and the values of the bank's securities after the run

The post–run value of equity. Let us assume that the bank has withstood the run of the repo creditors and now operates without repos in its liabilities. In this section we take (r_l, P_l) as given and study the properties of the equity value function[11]

$$U_0(c) := \sup_L U^{(0,P_l,L)}(c), \quad c \geq \underline{c}.$$

The formal derivation of the properties of U_0 that we discuss in the sequel may be found in AppendixA. Since the probability of liquidation is decreasing in the level of liquid reserves, the marginal value of liquid reserves decreases with c, as it is the case in several inventory models with pure equity financing (see e.g., Jeanblanc and Shiryaev (1995), Milne and Robertson (1996) and Décamps et al. (2011)). This implies that

[9] This is no longer true under payout restrictions imposed by the regulator (see 5.2).

[10] The presence of an *endogenous* tail risk, related to the use of repo funding, marks the key difference between our framework and the one considered by Bolton et al. (2014).

[11] Recall that, for the unregulated bank, $\underline{c} = 0$. Nevertheless, throughout this section we define the values of the contingent claims for any $\underline{c} \geq 0$, so as to exploit the obtained results in the setting where regulation is present.

the equity value function U_0, which is clearly increasing in the level of liquid reserves, is concave.[12] It is then optimal to retain earnings below a certain critical barrier b_0^* that corresponds to the level of liquid reserves for which the marginal value of an additional unit of retained earnings equals the marginal value of distributed dividends, i.e. $U_0'(b_0^*) = 1$.

In the *retention region* $(\underline{c}, b_0^*)$, the equity value function U_0 satisfies the ordinary differential equation

$$\rho U_0(c) = (1-\theta)^2 \frac{\sigma^2}{2} U_0''(c) + (1-\theta)(\mu - f_0) U_0'(c), \tag{3}$$

where $f_0 = r_l P_l + r_d P_d$ denotes the cost of debt financing. For each choice of r_l and P_l, Equation (3) has the general solution

$$A(r_l, P_l) e^{\beta_1 c} + B(r_l, P_l) e^{\beta_2 c},$$

where $\beta_1 = \beta_1(r_l, P_l) > 0$ and $\beta_2 = \beta_2(r_l, P_l) < 0$ are the roots of the characteristic polynomial of Equation (3).

Whenever the current level of liquid reserves c exceeds b_0^*, the shareholders' impatience outweighs their concerns regarding liquidation, and the difference $c - b_0^*$ is immediately distributed as dividends. In other words, in the region (b_0^*, ∞) the equity value function is affine:

$$U_0(c) = U_0(b_0^*) + c - b_0^*. \tag{4}$$

In order to guarantee the optimality of U_0 (our aim here is to maximize equity value), it must hold that U_0 is twice continuously differentiable. Combined with the fact that U_0 is linear on (b_0^*, ∞), this property yields an additional condition at the payout barrier, namely $U_0''(b_0^*) = 0$, which is commonly referred to as the *super–contact condition*[13].

Our strategy to find U_0 is to take an arbitrary level of liquid reserves $b_0 > 0$ and, using Equations (4) and (3) together with the conditions $U_0'(b_0) = 1$ and $U_0''(b_0) = 0$, to determine a candidate equity value function. The latter is given by

$$U_0(c; b_0) := \begin{cases} \frac{1}{\beta_1 - \beta_2}\Big(-\frac{\beta_2}{\beta_1} e^{\beta_1(c-b_0)} + \frac{\beta_1}{\beta_2} e^{\beta_2(c-b_0)}\Big), & \text{for } c \in [\underline{c}, b_0), \\ \frac{(1-\theta)(\mu - f_0)}{\rho} + c - b_0, & \text{for } c \geq b_0. \end{cases} \tag{5}$$

We show in AppendixB that, for r_l and P_l given, there exist a unique $b_0^* = b_0^*(r_l, P_l)$ that satisfies the equation

$$U_0(\underline{c}; b_0^*) = \tilde{u}_0(\underline{c}), \tag{6}$$

where $\tilde{u}_0 = \max\{\eta A + \underline{c} - P_d - P_l, 0\}$ is the liquidation value accruing to the shareholders in the event of liquidation. We also provide a verification theorem showing that $U_0(c; b_0^*)$ is, indeed, the optimal value function, i.e., the equation $U_0(c; b_0^*) = U_0(c)$ is satisfied for all $c \geq \underline{c}$. When $\underline{c} = 0$, Equation (6) has an explicit solution:

$$b_0^* = \frac{1}{\beta_1 - \beta_2} \log\Big(\frac{\beta_2}{\beta_1}\Big)^2.$$

The post–run value of the long–term debt. Let us now look at the market value of the long–term debt, which we denote by D_0 in this benchmark case. This value will depend critically on the payout strategy chosen by the bank's shareholders but, as long as r_l is assumed to be fixed, the long–term creditors are passive and take the dividend barrier as given. Therefore, we study the properties of D_0 for a fixed, arbitrary payout

[12] The concavity of the equity value function can be interpreted as a sort of "corporate" risk aversion arising from the risk of liquidation (see e.g., Milne and Robertson (1996) for a discussion on this phenomenon).
[13] We refer the reader to Dumas (1991) for a formal discussion on this property.

barrier b_0. By standard arguments, D_0 satisfies the following differential equation:

$$\rho D_0(c) = (1-\theta)^2 \frac{\sigma^2}{2} D_0''(c) + (1-\theta)(\mu - f_0) D_0'(c) + r_l P_l \quad \text{for} \quad c \in (\underline{c}, b_0). \tag{7}$$

In order to determine D_0, we need two boundary conditions: Since the optimal payout policy implies that the level of liquid reserves never exceeds b_0, the value of the long–term debt remains constant for any c that is greater than or equal to b_0. Thus, it must hold that $D_0'(b_0) = 0$. The second boundary condition is imposed at the liquidation threshold $\underline{c}$. Specifically, we have $D_0(\underline{c}) = \tilde{d}_0$, where $\tilde{d}_0 = \eta A (< P_l)$ if the long–term debt is senior to insured deposits and $\tilde{d}_0 = \eta A - P_d$ otherwise. Solving Equation (7) with the above–mentioned boundary conditions yields the following closed–form characterization of the market value of debt:

$$D_0(c) = \frac{r_l P_l}{\rho} - \left(\frac{r_l P_l}{\rho} - \tilde{d}_0\right) \left(\frac{e^{\beta_2 (c - \underline{c})} [\beta_1 e^{\beta_1 b} - \beta_2 e^{\beta_2 b}(1 - e^{(\beta_1 - \beta_2)\underline{c}})] - \beta_2 e^{\beta_1 c + \beta_2 (b - \underline{c})}}{\beta_1 e^{\beta_1 b} - \beta_2 e^{\beta_2 b + (\beta_1 - \beta_2)\underline{c}}} \right), \tag{8}$$

where the first term represents the perpetual value of the interest payments and the second term captures the impact of the liquidation and payout policies.

With the *after–run* values of equity and debt in hand, we are now in the position to define the values of the bank's securities and the optimal payout policy *before* the run.

3.2. The value of the bank's securities and the payout policy before the run

We now consider the optimal financing and payout policies when the bank has access to both long–term debt and repo financing. The main departure from the results obtained in Section 3.1 is due to the possibility of a run by the repo creditors. On the one hand, this allows for liquidation at levels of liquid reserves that are strictly higher than $\underline{c}$. On the other hand, conditional on the bank surviving a run, the dynamics of liquid reserves experience a *regime change*, since the cost of debt servicing per unit of time will be reduced by $r_s P_s dt$. These two facts notwithstanding, as long as a run on the repos does not occur, the optimal payout policy of the bank does not deviate significantly from that pertaining to the case considered in Section 3.1. More specifically, we show in AppendixB that given a debt structure (P_s, P_l) and an interest rate r_l, there exists a target level of liquid reserves $b_1^*(r_l, P_s, P_l) \geq 0$ such that, as long as there is no run on the repos, dividends are distributed so as to maintain the level of liquid reserves at or below b_1^*.[14]

The pre–run value of equity. As before, the value of b_1^* is closely related to the bank's equity value function. Given the pair (P_s, P_l), the latter is defined as

$$U_1(c) := \sup_L \; U^{(P_s, P_l, L)}(c).$$

We show in AppendixA that U_1 is a concave function of the level of liquid reserves for the same reasons as U_0. Moreover, U_1 is also affine beyond the pre–run dividend barrier. Namely, for $c > b_1^*$, we have

$$U_1(c) = U_1(b_1^*) + c - b_1^*.$$

However, for $c \in (\underline{c}, b_1^*)$, the characterization of U_1 strongly depends on the choice of the payout barrier b_1^*. Depending on the choice of the capital structure and the underlying parameters value, the bank may end up in one of three potential scenarios: If $b_1^* \in (\underline{c}, \underline{c} + P_s]$, the bank will always be liquidated in the case of a run by the repo creditors. In the case where $b_1^* \in (\underline{c} + P_s, \underline{c} + P_s + b_0^*]$, two outcomes are possible: If the liquid reserves suffice to absorb a loss caused by a creditors' run, i.e. if $c > \underline{c} + P_s$, then the bank survives, switches regime and follows the payout policy defined in Section 3.1. In contrast, if the run occurs when $c \in [\underline{c}, \underline{c} + P_s]$, then the bank is liquidated. Finally, if $b_1^* > \underline{c} + P_s + b_0^*$ and a run occurs while $c \in (\underline{c} + P_s + b_0^*, b_1^*]$, then

[14] For ease of reading, and as long as there are no grounds for confusion, we will refrain from writing the arguments of b_1^* in the text.

the bank makes a lump–sum payment of size $(c - P_s) - b_0^*$ immediately after the run, and then follows the optimal payout strategy defined in Section 3.1.

Since it is not possible to distinguish between these scenarios from an ex–ante perspective, one would be forced to characterize candidates for the optimal equity value function under each scenario and then perform a numerical analysis to see how the model's parameters affect the choice of b_1^*. In our analysis, however, we focus solely on the case where $b_1^* \in (\underline{c} + P_s, \underline{c} + P_s + b_0^*]$. On the one hand, this turns out to be the only case that manifests itself for the range of parameters that we use in our numerical simulations. On the other hand, the first and third scenarios are somehow pathological. Namely, neither setting a target reserves level that guarantees liquidation in the case of a run by the repo creditors, nor setting a target reserves level that opens the possibility of a lump–sum payment to shareholders after the run are particularly palatable from an economic perspective.

In order to simplify the presentation, let us introduce the following operator:

$$\mathcal{L}g := (1-\theta)^2 \frac{\sigma^2}{2} g'' + (1-\theta)(\mu - f_1) g' - \rho g,$$

where $f_1 = r_d P_d + r_l P_l + r_s P_s$ and g is any twice continuously differentiable function. If the condition $b_1^* \in (\underline{c} + P_s, \underline{c} + P_s + b_0^*]$ holds ex–post, then the equity value function U_1 satisfies the following system:

$$\mathcal{L}U_1(c) - \lambda[U_1(c) - \tilde{u}_1(c)] = 0, \quad c \in (\underline{c}, \underline{c} + P_s), \tag{9}$$

$$\mathcal{L}U_1(c) - \lambda[U_1(c) - U_0(c - P_s)] = 0, \quad c \in (\underline{c} + P_s, b_1^*), \tag{10}$$

$$U_1(c) - U_1(b_1^*) + b_1^* - c = 0, \quad c \geq b_1^*, \tag{11}$$

together with the boundary conditions $U_1'(b_1^*) = 1$ and $U_1''(b_1^*) = 0$. The *jump terms* $\lambda[U_1(c) - \tilde{u}_1(c)]$ and $\lambda[U_1(c) - U_0(c - P_s)]$ on the left–hand sides of Equations (9) and (10) reflect liquidation and a regime change, respectively. Here, the value accruing to shareholders in the case of liquidation caused by a repo creditors' run when $c \in (\underline{c}, \underline{c} + P_s)$ is given by $\tilde{u}_1(c) := \max\{\eta A + c - P_d - P_l - P_s, 0\}$, and the equity value U_0 after the run that leads to a regime change is as defined in Expression (5).

The closed–form solution to System (9) - (11) is cumbersome, so we relegate it to AppendixC.1. The value of equity at the optimal payout barrier b_1^*, however, is easy to obtain by inserting the boundary conditions $U_1'(b_1^*) = 1$ and $U_1''(b_1^*) = 0$ into Equation (10), which yields:

$$U_1(b_1^*) = \frac{(1-\theta)(\mu - f_1)}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda} U_0(b_1^* - P_s),$$

where the first term reflects the value of perpetual continuation and the second one captures the possibility of a regime switch in the event of a run.[15]

The pre–run value of debt. Let us now turn our attention to the value of the long–term, risky debt. The long–term creditors anticipate the impact of the rollover risk on the optimal liquidity–management policy of the bank. If a run takes place when the bank's level of liquid reserves is c and the bank survives, the market value of the long–term debt drops to $D_0(c - P_s)$, where D_0 is as defined in Expression (8). Alternatively, if the run by the repo creditors pushes the bank into liquidation, the long–term creditors will collect the liquidation value $\tilde{d}_1(c)$, which depends on the relative seniority between the risky debt and the insured deposits. Under these considerations, the market value of debt satisfies the following system:

$$\mathcal{L}D_1(c) + r_l P_l - \lambda[D_1(c) - \tilde{d}_1(c)] = 0, \quad c \in (\underline{c}, \underline{c} + P_s),$$

$$\mathcal{L}D_1(c) + r_l P_l - \lambda[D_1(c) - D_0(c - P_s)] = 0, \quad c \in (\underline{c} + P_s, b_1^*).$$

[15] Note that the effective discount rate, $\rho + \lambda$, is increasing with the probability of a run.

We solve the above system analytically in AppendixC.2, allowing for the two possible scenarios of priority structure of the long–term debt.

The pre–run payout barrier and the interest rate on the long–term debt. As a last step, we determine, for a given debt structure (P_s, P_l), an equilibrium interest rate on long–term debt r_l^* and the optimal dividend barrier b_1^*. Notice that the first–order conditions of the problem of maximizing the ex–ante value of the bank (see Expression (2)) with respect to the initial level of liquid reserves c_0 is $U_1'(c_0) = 1$. Yet, the same condition holds at the optimal payout barrier b_1^*, so that one must have $c_0 = b_1^*$.[16] In other words, it is optimal to establish the bank with the maximum level of liquid reserves so as to reduce the ex–ante probability of liquidation and, thereby, the cost of long–term debt. Therefore, for a given debt structure (P_s, P_l), the equilibrium interest rate on the long–term debt r_l^* and the optimal dividend barrier b_1^* are jointly determined by the system of equations[17]

$$D_1(b_1^*) = P_l \quad \text{and} \quad U_1(\underline{c}; b_1^*) = \tilde{u}_1(\underline{c}), \tag{12}$$

where $\tilde{u}_1(\underline{c})$ is the amount accruing to the shareholders at liquidation. By establishing an explicit link between the optimal payout policy and the financing structure, the above system functions as the vehicle via which the rollover risk pertaining to the repo financing will affect the optimal financing and payout decisions.

It is a priori not clear whether or not the pre–run payout barrier b_1^* exceeds the payout barrier b_0^* that the shareholders would choose after the run of the repo creditors, should the bank survive it. On the one hand, after a run the bank is no longer exposed to the rollover risk, which reduces its precautionary motives for holding liquid reserves. This may suggest that it is optimal to reduce the target reserves level after the run. On the other hand, the bank becomes more profitable in expectation, since it no longer has to make payments to the repo creditors. This generates a higher franchise–value effect and may induce the bank to hold more liquid reserves. Which of these two effects dominates will become apparent from the numerical analysis conducted in Section 4.

Applying the results of our analysis to the shareholders' optimization Problem (2), one can easily see that the latter reduces to the choice of the optimal financing structure:

$$\max_{(P_s,P_l)\in\Pi(c)} \quad V_1^*(P_s, P_l) = \Big\{U_1(b_1^*) - b_1^* - (I - P_l - P_s - P_d)\Big\}.$$

In the sequel we solve the shareholders' optimization problem numerically, using the following parameter values: the risk–free rate $\rho = 5\%$, the cost of insured deposits $r_d = 4.5\%$, the cost of repo funding $r = 2.5\%$, the expected return on risky assets $\mu = \{20\%, 25\%, 30\%\}$, the volatility of the pre–tax earnings $\sigma = 18\%$, the tax rate $\theta = 35\%$, the intensity of the repo–funding withdrawal $\lambda = \{0.03, 0.05, 0.07\}$, the proportion of assets that can serve as collateral $\eta = 0.3$ and the book asset value equal to the first–best value $A = (1-\theta)\mu/\rho$.[18]

[16] This property holds due to the absence of the proportional equity–issuance costs. If we allowed for equity issuance with proportional deadweight costs, it would be optimal to establish the bank with an initial level of liquid reserves $c_0 < b_1^*$.

[17] The caveat that the non–linear equation $D_1(b_1^*) = P_l$ may have multiple roots is taken into account when we search for numerical solutions in the next section. Unreported numerical results show that the value of the bank evaluated at the payout barrier is decreasing over the range of admissible values of the interest rate. Thus, r_l^* is always given by the smallest root of $D_1(b_1^*) = P_l$.

[18] The choice of parameters in our numerical exercise is partially motivated by empirical evidence and partially done so as to have an interior solution to the problem of finding the optimal level of repo funding. In particular, the value of r_s is chosen close to the average for the 1–year LIBOR computed for the period 2000–2014. The value of η is chosen close to the estimations of Gropp and Heider (2010), reporting that a bank's collateral in average amounts to 27% of its assets. At the same time, the model turns out to be highly sensitive to the choice of the parameters λ and σ. Namely, the bank has no incentives to use any repo funding when σ is relatively low and/or when λ is relatively high.

4. The Liability Structure of an Unregulated Bank

In order to understand better what drives a bank's choice between stable but costlier long–term debt and unstable but relatively cheap repo funding, we first solve the shareholders' problem for a hypothetical bank that has no access to insured deposits. Then, we introduce deposit financing and investigate the impact of the relative priority between the insured deposits and the risky debt on the bank's liability structure.

4.1. No access to deposit funding

On the first stage of our numerical analysis, we disregard the possibility of deposit financing and solve the shareholders' problem for $P_d = 0$. To illustrate the impact of the repo funding, we also evaluate the optimal financing and payout policies in the setting in which the bank has no access to it (we refer to this setting as *the benchmark*).[19] The optimal characteristics of the bank's financing and payout policies are reported in Table 1.

The first observation to be made in regard to the numerical results presented in Table 1 is that the presence of repo funding in the bank's liability structure impairs its ability to raise long–term debt. However, the use of repo funding increases both the overall leverage and the ex–ante value of the bank. Interestingly, banks with lower expected returns on risky assets benefit more from the value–increasing effect generated by the presence of repo funding in their financing structure. In particular, in our numerical example, the use of short–term, secured funding by the bank with $\mu = 20\%$ results in a 13% increase in the ex–ante value of the bank relative to the benchmark case, whereas for the bank with $\mu = 30\%$ the benefits of using repo funding are marginal and amount only to a 2% increase in the ex–ante bank value (this comparison is made for $\lambda = 0.03$).

An important fact to be noted is that the amount of repo funding optimally chosen by banks in the absence of insured deposits is relatively low and the collateral constraint $P_s \leq \eta A$ is far from binding. Thus, contrary to the anecdotal and empirical evidence on banks' over–reliance on repo funding, our analysis shows that, when placed in the position of a non–financial firm, a bank would be better off abstaining from an aggressive use of repo funding. This suggests that the increased reliance on repo funding observed in practice might be partly rooted in the distortions induced by the access to (insured) deposit funding (we explore this conjecture below). The reason why the bank has no incentives to aggressively rely on repo funding is tied to the fact that the presence of the latter magnifies the cost of its long–term debt, which we refer to as the *shadow cost* of repo funding. To illustrate this effect, we compute the shadow cost of repo funding as the difference between the equilibrium interest rate on the long–term debt obtained in the setting in which the bank has access to repo funding and the equilibrium interest rate obtained in the benchmark case:

$$\textit{Shadow cost} = r_l^*(P_s, P_l) - r_l^*(P_l). \tag{13}$$

The left–hand panel of Figure 1 illustrates the typical patterns of the shadow cost of repo funding, computed for any given level of long–term debt P_l for two different levels of expected returns on risky assets. This shadow cost can be interpreted as a kind of premium rewarding long–term creditors for the negative externalities imposed on them by the bank's decision to use repo funding. It is the very presence of this shadow cost that curbs the bank's appetite for "cheap", short–term funding. The difference in the magnitude of the shadow cost of repo funding for banks with different levels of expected return on risky asset can be easily understood by taking into account the effect of repo funding on the optimal payout policy. This effect is illustrated in the right–hand panel of Figure 1, which depicts the upward adjustments that the bank would make to its target level of liquid reserves relative to those it would set when making no use of repo funding.

[19] The values of equity and debt in this setting correspond to the functions U_0 and D_0 defined in Section 3.1, whereas the optimal payout barrier and the interest rate on long–term debt result from jointly solving the equations $D_0(b_0^*) = P_l$ and $U_0(0; b_0^*) = 0$.

Table 1: The optimal financing and payout policies in the absence of insured deposits

	$\lambda = 0.03$			$\lambda = 0.05$			$\lambda = 0.07$			Benchmark: $P_s = 0$		
	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$
Bank value at initiation, $V_1^*(P_s, P_l)$	0.119	0.376	0.649	0.109	0.366	0.640	0.106	0.364	0.637	0.106	0.363	0.637
Impact of ST debt on value	13.19%	3.59%	1.93%	3.67%	1%	0.54%	0.85%	0.23%	0.12%	-	-	-
The value of risky assets, A	2.6	3.25	3.9	2.6	3.25	3.9	2.6	3.25	3.9	2.6	3.25	3.9
Pleadgeable assets, ηA	0.78	0.98	1.17	0.78	0.98	1.17	0.78	0.98	1.17	0.78	0.98	1.17
Secured ST debt, P_s^*	0.159	0.145	0.135	0.087	0.08	0.074	0.042	0.039	0.036	-	-	-
Risky LT debt, P_l^*	1.946	2.728	3.544	1.999	2.775	3.589	2.029	2.803	3.614	2.05	2.83	3.63
Total debt, $P_s^* + P_l^*$	2.105	2.873	3.679	2.086	2.855	3.663	2.071	2.842	3.65	2.05	2.83	3.63
ST debt/Total Debt	7.55%	5.05%	3.67%	4.17%	2.8%	2.02%	2.03%	1.37%	0.99%	-	-	-
ST debt/Pledgeable assets	20.38%	14.87%	11.54%	11.15%	8.21%	6.32%	5.38%	4%	3.08%	-	-	-
LT interest rate, r_l^*	5.477%	5.388%	5.331%	5.467%	5.381%	5.326%	5.462%	5.379%	5.324%	5.457%	5.377%	5.32%
Pre–run target liquid reserves, b_1^*	0.517	0.5113	0.5045	0.5063	0.5022	0.4962	0.4985	0.4951	0.4898	-	-	-
Post-run target liquid reserves, b_0^*	0.4892	0.485	0.4798	0.4899	0.4862	0.4813	0.4902	0.487	0.4822	0.4903	0.4875	0.4826
$(b_1^* - b_0^*)/b_0^*$	5.67%	5.43%	5.15%	3.36%	3.27%	3.08%	1.69%	1.68%	1.59%	-	-	-
Book Leverage	67.53%	76.38%	83.53%	67.15%	76.09%	83.32%	66.84%	75.88%	83.15%	-	-	-
ST debt/Total liabilities	5.1%	3.86%	3.07%	2.8%	2.13%	1.68%	1.36%	1.04%	0.82%	-	-	-
LT debt/Total liabilities	62.43%	72.53%	80.46%	64.35%	73.96%	81.64%	65.48%	74.84%	82.33%	64.15%	68.9%	72.32%
Equity/Total liabilities	32.47%	23.62%	16.47%	32.85%	23.91%	16.68%	33.16%	24.12%	16.85%	35.85%	31.1%	27.68%

Notes: This table reports the optimal financing and payout decisions of a bank that has no access to insured deposits. The last three columns characterize the bank's optimal policies in the setting in which the bank has no access to repo funding (benchmark). The optimal payout barrier and the interest rate on the long–term debt in the benchmark case are obtained by jointly solving the equations $D_0(b_0^*) = P_l$ and $U_0(0; b_0^*) = 0$.

These adjustments are computed according to the formula

$$\frac{b_1^*(P_s, P_l) - b_0^*(P_l)}{b_0^*(P_l)} \times 100\%. \tag{14}$$

It turns out that, for the same debt structure (P_s, P_l), a bank enjoying higher expected return on risky assets would be able to make a larger upward adjustment of its target level of liquid reserves, thereby being able to offset a larger fraction of the additional liquidation risk related to the use of repo funding. As a result, banks with higher μ's face a lower shadow cost of repo funding.

Result 1. *The use of repos generates an additional component in the spreads of the long–term risky debt, which decreases with the bank expected returns on risky assets.*

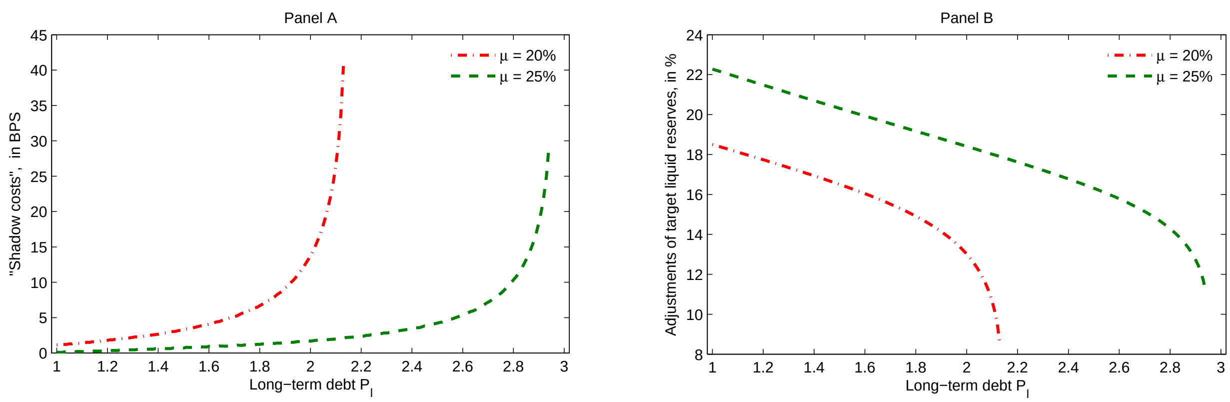

Figure 1: The shadow cost of repo funding

Notes: This figure illustrates the shadow–cost component of the long–term debt spreads (Panel A, in bps) and the scale of the upward adjustments in the target level of liquid reserves caused by the use of repo funding (Panel B, in %). For this example we used $P_s = 0.20$ and $\lambda = 0.05$.

The expected returns on risky assets appear to be an important driver of the debt–structure choice. Table 1 shows that the bank with a lower μ (i.e., the low–profit bank) exhibits a higher ratio of repo funding to total debt. The reason is that low–profit banks face a higher wedge between the costs of repo and long–term debt financing. Indeed, lower expected returns imply higher liquidation risks and, thus, higher costs of long–term financing. This initial cost disadvantage of the long–term debt for low profit banks is reinforced by the shadow–cost effect. As it was pointed out above, lower expected returns on risky assets undermine the bank's ability to maintain strong liquidity buffers to hedge against the rollover risk related to the use of repo funding. The higher shadow cost that low–profit banks would face when raising the same amount of long–term debt as the high–profit banks, would exacerbate the relative cost disadvantage of the long–term debt even further; thus, pushing the low–profit banks to rely more on repo funding. Therefore, our analysis suggests that repos play a more prominent role in the financing of banks that do not benefit from a high cash–flow from their risky assets.[20]

Result 2. *Repo funding plays a more prominent role in the financing structure of banks with lower returns on risky assets.*

[20] One can interpret an extensive use of repo funding as taking on tail risk on the liability side of the balance sheet. Viewed in this light, the effect of the expected returns on risky assets in our setting is akin to the effect of the franchise value widely discussed in the banking literature.

Another interesting observation that can be drawn from Table 1 is that both the interest rate on long–term debt and the target level of liquid reserves that the bank would fix prior to the run are *decreasing* with the intensity λ of the repo creditors' runs. This counterintuitive result is due to the fact that, when estimating rollover risk to be low, the bank would be tempted to use a larger volume of repo funding. This would require setting a higher target level of liquid reserves in order to alleviate the consequent increase in the liquidation risk.[21] Yet, a larger volume of repo funding also reduces the expected residual value accruing to the long–term debt creditors in the case of liquidation. This adverse effect of repo financing cannot be completely offset via the adjustment of the optimal payout policy (since maintaining high liquidity buffers is costly) and will, thus, translate into higher costs of long–term debt. This positive relation between the cost of long–term debt and the proportion of repos that emerged from our numerical analysis echoes the empirical evidence documented by Valenzuela (2013) and Gopalan et al. (2013) for non–financial firms.

Finally, notice that, both for highly–profitable and lowly–profitable banks, the pre–run target level of liquid reserves, b_1^*, exceeds the target level b_0^* that the bank would set if it were to survive the run. The wedge between these two target levels of liquid reserves can be seen as a kind of precautionary liquidity buffer needed both to protect the bank against the rollover risk inherent in the use of repo funding and to mitigate the shadow–cost component of the long–term debt spreads. This outcome resonates with the empirical evidence documented by Harford et al. (2014) for non–financial firms, namely, that firms tend to strengthen their liquid reserves in order to manage their refinancing risk better.

4.2. Deposit funding and the depositor preference rule

Having considered the optimal financing and payout decisions of a bank without access to insured deposits, we now explore how the access to deposit funding, *under mis–priced deposit guarantees* (recall that $r_d < \rho$ is exogenous), affects the choice of the non–deposit sources of funding. We distinguish between two scenarios of priority rules. In the first scenario, deposits are junior to long–term debt. The second scenario mirrors the U.S. depositor preference law introduced on a national level in 1993, according to which domestic deposits are senior to other debt claims.[22]

We solve the shareholders' optimization problem for the volume of insured deposits $P_d = 2$, which accounts for $45 - -75\%$ of total assets in our numerical example (conditional on the values of μ and λ in the different scenarios), and report the results in Table 2. A comparison of the bank's optimal financing and payout decisions with those obtained in the setting with no deposit funding reveals a number of interesting results.

First, it turns out that a higher volume of deposit funding increases the relative importance of repos, which is reflected in the ratios $P_s^*/(P_s^* + P_l^*)$ and $P_s^*/(A + b_1^*)$. This feature obeys the fact that, when the supply of deposit funding is taken as given, a larger volume of deposit funding entails a reduction in the effective cash–flow rate. The impact of the access to deposit funding on the choice of the non–deposit funding structure is, therefore, similar to the effect of lower expected returns on risky assets that we discussed in Section 4.1; namely, banks tend to substitute a larger proportion of thier long–term debt by repo funding.

Second, making insured deposits senior to long–term debt renders banks even more reliant on repo funding. The reason is that the depositor preference rule reduces the amount accruing to the long–term creditors in the case of liquidation, which translates into higher costs of long–term debt and, thus, exacerbates the relative cost advantage of repo funding.

Result 3. *Abundant deposit funding and the depositor seniority rule exacerbate a bank's reliance on repo funding.*

Notice, however, that the quantitative impacts of deposit funding and deposit seniority rules on the optimal volume of repo funding remain relatively modest for most combinations of input parameters. In

[21]The adjustment of the target level of liquid reserves, actually, explains why the quantitative effect of the run intensity on the cost of long–term debt is marginal.

[22]The introduction of a similar law for European countries is being currently considered by the European authorities.

other words, claiming that abundant deposit funding is the only major factor that pushes the bank to *aggressively* substitute long–term debt by repo funding would be inappropriate. The reason being that the incentives to increase the reliance on repo funding are still mitigated by the potential shadow cost.

5. The Impact of Regulation

So far, our analysis has been focused on the policy decisions of an unregulated bank. In reality, however, banks face regulatory requirements. In this section, we try to understand how various regulatory measures affect a bank's financing structure and, in particular, the relative importance of repos within a bank'a liabilities.

5.1. Liquidity regulation

We start by exploring the effect of liquidity regulation on the optimal payout and financing policies of a bank. Given that the (over)reliance of banks on repo funding is commonly recognized as an important source of financial instability, one of the objectives of the new liquidity regulation introduced by Basel III is to reduce the use of repo funding by banks (see e.g. IMF Global Financial Stability Report (2013), Chapter 3). We introduce liquidity regulation in the spirit of the Basel III Liquidity Coverage Ratio (LCR). This regulation stipulates that banks must maintain a level of highly liquid assets in a certain proportion to the volume of fund withdrawals expected in the next 30 days. In our setting, this idea is captured by the assumption that the level of a bank's liquid reserves must always exceed a certain fraction of its repo funding. Hence, liquidity regulation in our model establishes a link between the volume of repo funding and the liquidation threshold:

$$\underline{c}^*(\Lambda) = \Lambda P_s,$$

where $\Lambda \geq 0$ is a regulatory parameter reflecting the tightness of the liquidity requirements.

When solving their optimization problem under such a regulatory constraint, the bank's shareholders must take into account the feedback effect that their choice of P_s would produce on the liquidation and payout policies. Intuitively, one would expect that tighter liquidity requirements should curb the bank's appetite for repo funding and should reduce banks' exposure to the run–triggered liquidation risk. To verify this conjecture, we turn to a numerical analysis.

The left–hand panel of Figure 2 depicts the proportions of repos and long–term debt in the bank's total financing structure as functions of Λ. Indeed, when faced with tighter liquidity requirements, the bank reduces its reliance on repo funding and can even be completely discouraged from its use. Simultaneously, the bank increases its reliance on long–term debt funding. In other words, imposing tighter liquidity regulation leads to substitution of repos by long–term debt funding.

To understand the underlying mechanism at work, notice that, given any fixed P_s, the presence of liquidity requirements translates into a strictly positive liquidation threshold; hence, it increases the risk of liquidation. In principle, the bank could offset this adverse effect by making an upward adjustment on its target level of liquid reserves. Yet, due to the deadweight cost of holding liquidity, the bank would prefer to reduce its reliance on repos rather than to substantially strengthen its liquidity buffer. The reduction in the level of repo funding reduces the cost of long–term debt (via the shadow–cost channel), which eventually enables the bank to increase its reliance on long–term debt.

Result 4. *Tightening liquidity requirements induces substitution of repo funding by long–term debt funding.*

Given that tightening liquidity requirements generates a substitution effect, it is worthwhile to consider the impact of liquidity requirements on the bank's liquidation probability. In AppendixD we describe a methodology that enables us to compute the liquidation probability, allowing for the possibility of a run by the repo creditors. We apply this methodology to evaluate the relative change in the liquidation probability caused by the implementation of liquidity regulation, which is computed according to the following formula:

$$\Delta P(T, c; \Lambda) = \frac{P(T, c; \Lambda) - P(T, c; 0)}{P(T, c; 0)},$$

Table 2: The impact of deposit funding and seniority rules

			P_s^*	P_l^*	$(P_s^* + P_l^*)$	$P_s^*/(P_s^* + P_l^*)$	b_1^*	b_0^*	r_l	Market lev.	Book lev.	V_1^*	f_1	$P_s^*/(A + b_1^*)$
$\lambda = 0.03$	$\mu = 20\%$	$P_d = 0$	0.159	1.946	2.105	7.55%	0.517	0.4892	5.48%	0.65	0.68	0.119	5.81%	5.10%
		$P_d = 2$, junior	0.214	0.862	1.076	19.89%	0.4781	0.4721	5.56%	0.82	1	0.678	3.69%	6.95%
		$P_d = 2$, senior	0.229	0.524	0.753	30.41%	0.5246	0.4882	6.36%	0.76	0.88	0.501	4.61%	7.33%
	$\mu = 25\%$	$P_d = 0$	0.145	2.728	2.873	5.05%	0.5113	0.485	5.39%	0.69	0.76	0.376	6.46%	3.86%
		$P_d = 2$, junior	0.184	1.489	1.673	11%	0.5145	0.4889	5.49%	0.8	0.98	0.827	4.79%	4.89%
		$P_d = 2$, senior	0.183	1.167	1.350	13.56%	0.5247	0.4895	5.79%	0.75	0.89	0.682	5.71%	4.85%
	$\mu = 30\%$	$P_d = 0$	0.130	3.550	3.680	3.53%	0.5031	0.48	5.33%	0.73	0.84	0.649	6.99%	2.95%
		$P_d = 2$, junior	0.163	2.204	2.367	6.89%	0.5177	0.4899	5.41%	0.8	0.99	1.045	5.64%	3.69%
		$P_d = 2$, senior	0.159	1.895	2.054	7.74%	0.5155	0.4842	5.57%	0.76	0.92	0.915	6.53%	3.60%
$\lambda = 0.05$	$\mu = 20\%$	$P_d = 0$	0.087	1.999	2.086	4.17%	0.5063	0.4899	5.47%	0.65	0.67	0.109	5.76%	2.80%
		$P_d = 2$, junior	0.101	0.956	1.057	9.56%	0.4608	0.4607	5.57%	0.82	1	0.666	3.52%	3.30%
		$P_d = 2$, senior	0.150	0.563	0.713	21.04%	0.5159	0.487	6.29%	0.75	0.87	0.48	4.6%	4.81%
	$\mu = 25\%$	$P_d = 0$	0.080	2.775	2.855	2.8%	0.5022	0.4862	5.38%	0.69	0.76	0.366	6.41%	2.13%
		$P_d = 2$, junior	0.097	1.555	1.652	5.87%	0.501	0.4871	5.48%	0.8	0.97	0.816	4.7%	2.59%
		$P_d = 2$, senior	0.120	1.200	1.320	9.09%	0.517	0.4899	5.77%	0.75	0.88	0.667	5.71%	3.19%
	$\mu = 30\%$	$P_d = 0$	0.074	3.589	3.663	2.02%	0.4962	0.4813	5.33%	0.73	0.83	0.64	6.96%	1.68%
		$P_d = 2$, junior	0.088	2.259	2.347	3.75%	0.5065	0.4903	5.40%	0.8	0.99	1.035	5.58%	2%
		$P_d = 2$, senior	0.100	1.929	2.029	4.93%	0.5077	0.4851	5.56%	0.76	0.91	0.902	6.52%	2.27%
$\lambda = 0.07$	$\mu = 20\%$	$P_d = 0$	0.042	2.029	2.071	2.03%	0.4985	0.4902	5.46%	0.65	0.67	0.106	5.73%	1.36%
		$P_d = 2$, junior	0.041	1.003	1.044	3.93%	0.4524	0.4534	5.58%	0.82	1	0.663	3.44%	1.34%
		$P_d = 2$, senior	0.106	0.585	0.691	15.34%	0.5094	0.4861	6.26%	0.75	0.87	0.473	4.6%	3.41%
	$\mu = 25\%$	$P_d = 0$	0.040	2.802	2.842	1.41%	0.4954	0.4869	5.38%	0.69	0.76	0.364	6.39%	1.07%
		$P_d = 2$, junior	0.046	1.591	1.637	2.81%	0.4924	0.4857	5.48%	0.8	0.97	0.812	4.66%	1.23%
		$P_d = 2$, senior	0.077	1.224	1.301	5.92%	0.5089	0.4901	5.76%	0.75	0.88	0.662	5.69%	2.05%
	$\mu = 30\%$	$P_d = 0$	0.036	3.614	3.650	0.99%	0.4898	0.4822	5.32%	0.73	0.83	0.637	6.94%	0.82%
		$P_d = 2$, junior	0.043	2.290	2.333	1.84%	0.4987	0.4904	5.40%	0.80	0.99	1.032	5.55%	0.98%
		$P_d = 2$, senior	0.062	1.951	2.013	3.08%	0.5009	0.4857	5.55%	0.76	0.91	0.898	6.51%	1.41%

Notes: In this table we report the impact of (mis–priced) deposit funding and the seniority rules on a bank's financing and payout policies for different combinations of μ and λ. These results illustrate that: (i) the access to deposit funding and the depositor priority rule induce a higher reliance on repo funding (this feature is reflected in the ratios $P_s^*/(P_s^* + P_l^*)$ and $P_s^*/(A + b_1^*)$ and manifests itself for most outcomes); (ii) the access to deposit funding exacerbates leverage, reduces the effective average profitability f_1 and increases the bank's value V_1^*; (iii) the implementation of the depositor seniority rule reduces leverage, increases the effective average profitability f_1 and reduces the bank's value V_1^*. The reduction in leverage induced by the implementation of the depositor preference rule is consistent with the theoretical predictions of Hugonnier and Morellec (2014) and the empirical evidence documented by Danisewicz et al. (2015).

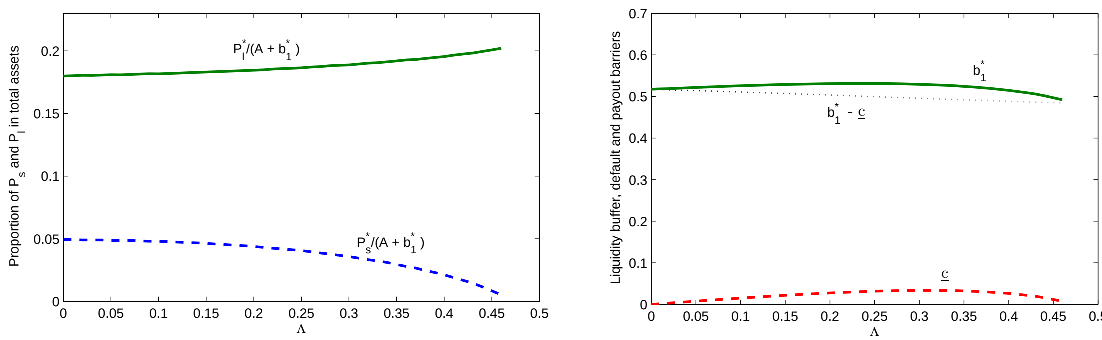

Figure 2: Liquidity requirements and bank policies

Notes: The left–hand panel depicts the proportions of repos and long–term debt in the overall financing structure. The parameter values are: $P_d = 2$, $\rho = 5\%$, $r_d = 4.5\%$, $r_s = 2.5\%$, $\mu = 20\%$, $\sigma = 18\%$, $\theta = 35\%$, $\eta = 0.3$ and $\lambda = 0.05$. The bank makes no use of repo funding for $\Lambda > 0.45$. The right–hand panel reports the optimal payout and liquidation barriers. Notice that the liquidity buffer $b_1^* - \underline{c}$ (it is represented by the dotted line) is monotonically decreasing in Λ.

where $P(T, c; \Lambda)$ is the probability of liquidation defined in AppendixD and T is a time horizon.

We set $T = 1$ (1 year) and first compute the relative changes in the liquidation probability evaluated at the target level of liquid reserves, i.e. $\Delta P(T, b_1^*(\Lambda); \Lambda)$. The tiny magnitude of these changes, reported in the left–hand panel of Figure 3 (in percentage points), suggests that the liquidation probability at the target level of liquid reserves is insensitive to the changes in the bank policies caused by liquidity regulation. However, the impact of liquidity regulation becomes non–negligible at levels of liquid reserves that deviate from the target. To see this effect, we compute $\Delta P(T, \underline{c}(\Lambda) + \alpha; \Lambda)$ where the parameter α reflecting the size of the liquidity buffer is such that $0 < \alpha \leq \min\{b_1^*(\Lambda) - \underline{c}^*(\Lambda)\}_{\Lambda \in (0,1)}$. The numerical results reported in the right–hand panel of Figure 3 show that tighter liquidity regulation, in fact, reduces the liquidation probability despite the debt substitution effect that we discussed previously.

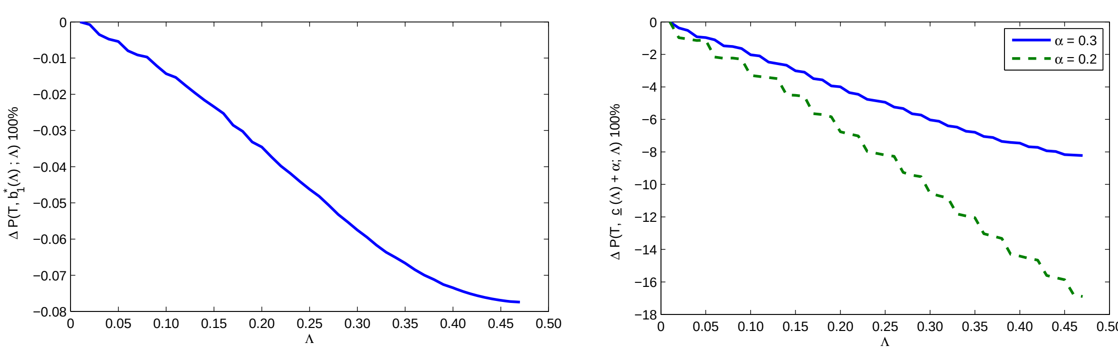

Figure 3: Relative changes in the bank's liquidation probability

Notes: This figure depicts the relative changes in the liquidation probability at different levels of liquid reserves for a 1–year time horizon. The numerical method used to compute the liquidation probability behaves more regularly for larger values of α.

Finally, we analyze the impact of liquidity regulation on the bank's leverage and value (see Figure 4). It turns out that tightening the liquidity requirements reduces the overall bank leverage and, consequently, the overall bank value. Both effects, however, remain relatively modest due to the partial substitution of repo funding by long–term debt.

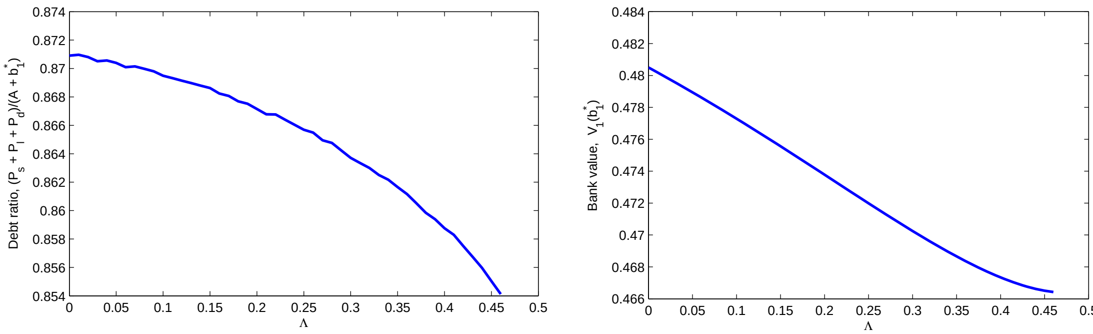

Figure 4: Debt ratio and bank value

Notes: This figure depicts the debt ratio and bank value as functions of Λ. For $\Lambda = 0.45$, the debt ratio is reduced by almost 2% with respect to the unregulated case ($\Lambda = 0$) and the bank value declines by almost 3%.

5.2. Payout restrictions

One more novel feature of the Basel III regulation was the introduction of payout restrictions on insufficiently capitalized banks.[23] A question of interest is how these payout restrictions affect the banks' ex–ante choices of financing structure and, in particular, the relative importance of repo financing. To explore this question, we study below the case where bank shareholders are prohibited from distributing dividends when the book value of bank equity falls below a certain critical level k_{div}, i.e. when

$$\frac{A + C^{\pi}(t) - (P_s \mathbb{1}_{\{t \leq \tau^*\}} + P_l + P_d)}{A + C^{\pi}(t)} < k_{div}. \tag{15}$$

For a given liability structure (P_s, P_l, P_d), Condition (15) can be rewritten in terms of a critical level of liquid reserves below which dividend distribution is forbidden. Specifically, the regulatory payout threshold before the run is given by

$$b_1^{reg}(P_s, P_l, P_d) = \max\Big\{\frac{P_s + P_l + P_d}{1 - k_{div}} - A, 0\Big\},$$

whereas the regulatory payout threshold after the run is

$$b_0^{reg}(P_l, P_d) = \max\Big\{\frac{P_l + P_d}{1 - k_{div}} - A, 0\Big\}.$$

To understand how payout restrictions affect the bank's financing policy, let us fix an arbitrary volume P_s of repos and consider the shareholders' optimization problem, which now consists of choosing the initial level of liquid reserves c_0, the volume of long–term debt P_l and the optimal payout strategy characterized by the pair of payout thresholds (b_0, b_1).

First, let us take an arbitrary debt structure (P_l, r_l) and consider the choices of the pre–run payout barrier and the initial level of liquid reserves. As we have seen in Section 3, without regulation, the optimal pre–run payout barrier was characterized by the relation $U_1(0; b_1^*) = \tilde{u}_1(0)$ and the shareholders initiated the bank with the level of liquid reserves $c_0 = b_1^*$. In the regulated environment, two cases are possible. In the first case, $b_1^{reg} \leq b_1^*$ and the payout restrictions have no impact on the bank's policies. In the alternative case ($b_1^{reg} > b_1^*$), the bank is be forced to abstain from distributing dividends when its liquid reserves are below b_1^{reg}. This reduces the value of the shareholders' equity and affects the marginal value of liquid reserves. Recall that, in the absence of payout restrictions, the latter was monotonically decreasing in the level of c,

[23] Specifically, payout restrictions apply to the banks that fail to meet the capital conservation buffer requirement (the requirement to maintain 2.5% of common Tier equity capital on top of minimum capital requirements).

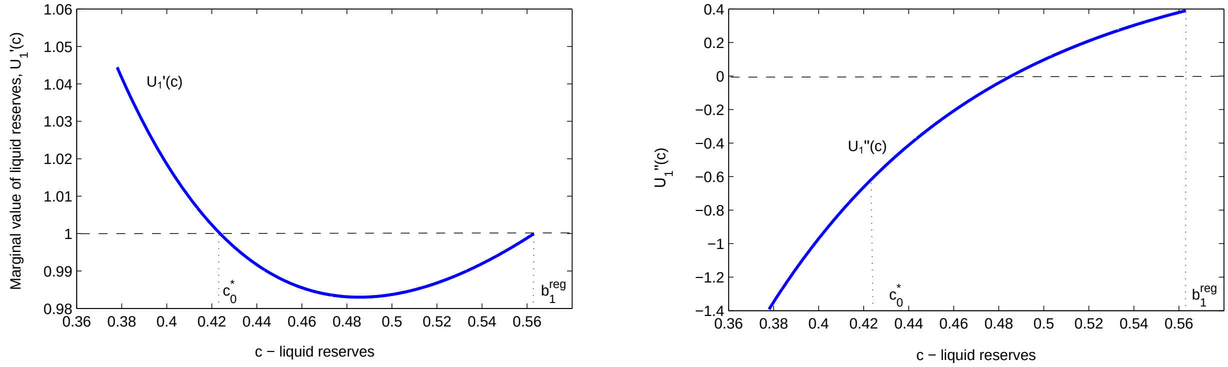

Figure 5: The impact of payout restrictions on the marginal value of c

i.e. $U_1''(c) < 0$ over $(0, b_1)$, and $U_1''(b_1) = 0$. This is, however, no longer the case when the payout barrier is set to b_1^{reg}. In Figure 5 we plot the typical patterns of the marginal value of liquid reserves U_1' and its first derivative in the neighborhood of mandatory payout barrier $b_1^{reg} > b_1^*$. This shows that, in the neighborhood of b_1^{reg}, the equity value U_1 becomes *locally convex* and there exists a certain level of liquid reserves $c_0^* < b_1^{reg}$, such that $U_1'(c_0^*) = 1$ and $U_1''(c_0^*) < 0$.

It is easy to see that $c_0^* = \arg\max\{U_1^{\pi}(c_0) - c_0\}$. Thus, it represents the optimal level of liquid reserves that shareholders will choose to initiate the bank. As we have seen in Section 3, in the absence of payout restrictions, c_0^* coincides with the target level of liquid reserves b_1^*. However, payout restrictions reduce the expected value of dividends; hence, shareholders make lower initial equity contributions compared to the unregulated case, i.e. $c_0^* < b_1^*$. The formal proof of this property can be found in AppendixE.2.

Result 5. *When $b_1^{reg} > b_1^*$, the shareholders choose to initiate the bank with a level of liquid reserves $c_0^* < b_1^*$.*

Consider now the impact of payout restrictions on the cost of long–term debt, which is determined by the condition $D_1(c_0^*) = P_l$. The presence of payout restrictions generates two counteracting effects. On the one hand, a lower initial capitalization (as compared to the unregulated case) increases the bank's liquidation probability for short time horizons, which in turn reduces the market value of debt. On the other hand, the probability of liquidation for long time horizons decreases, since the bank will be forced to maintain a larger liquidity buffer. Numerical simulations show that the latter effect dominates when the face value of long–term debt is not too high. In such a case, for any given level of P_s, the bank would choose a *higher* level of long–term debt than in the unregulated case, which would in turn *increase* its ex–ante value. Even though the value–increasing effect generated by the payout restrictions may seem counterintuitive at a first glance, it admits a very natural explanation. In fact, in the absence of the payout restrictions, shareholders do worse due to a commitment problem. Indeed, even though delaying dividend payment, i.e. setting a higher payout threshold, may be optimal from an ex–ante perspective because it reduces the cost of debt financing and, thus, increases the bank's value, once the long–term debt is issued, the shareholders would have incentives to switch to the payout policy that maximizes their dividend payoffs and implies a higher liquidation risk. Rational creditors would anticipate this behavior and demand a higher interest rate on their debt. In contrast, the payout restrictions imposed by the regulator work as a credible commitment mechanism that helps the bank reduce its cost of long–term financing.

The reduction in the cost of long–term debt financing caused by the presence of payout restrictions will in turn reduce the cost advantage of repos, thereby inducing the bank to substitute such funding by long–term debt one. Table 3 illustrates the effects of the payout restrictions on the bank's financing and payout policies.

One observes that payout restrictions result in a substantially lower volume of repos and a lower cost of long–term debt due to the aforementioned effect.

Table 3: Impact of payout restrictions

	k_{div}	P_s	P_l	$\frac{P_s}{P_s+P_l}$	c_0^*	b_1	b_0	r_l	MLR	BLR	V_1^*
$\lambda = 0.03$	–	0.23	0.52	0.31	0.5256	0.5256	0.4883	0.0635	0.76	0.88	0.5012
	0.12	0.08	0.75	0.10	0.3773	0.6159	0.5250	0.0599	0.81	0.88	0.5259
	0.14	0.09	0.69	0.12	0.3835	0.6326	0.5279	0.0582	0.79	0.86	0.5202
	0.16	0.09	0.63	0.13	0.3903	0.6381	0.5310	0.0569	0.78	0.84	0.5099
$\lambda = 0.05$	–	0.15	0.56	0.21	0.5163	0.5163	0.4871	0.0628	0.75	0.87	0.4805
	0.12	0.03	0.78	0.04	0.3847	0.5932	0.5591	0.0594	0.8	0.88	0.5194
	0.14	0.04	0.72	0.05	0.3903	0.6093	0.5628	0.0579	0.79	0.86	0.5138
	0.16	0.05	0.65	0.07	0.4003	0.6143	0.5548	0.0568	0.77	0.84	0.5039

Notes: This table illustrates the impact of payout restrictions on the bank's policies. The parameter values are: $\rho = 5\%$, $r_d = 4.5\%$, $r = 2.5\%$, $\mu = 20\%$, $\sigma = 18\%$, $\theta = 35\%$, $P_d = 2$ and $\eta = 0.3$. The bank value reported in the last column declines with k_{div} because the reduction in the value of equity dominates the benefits generated by the lower costs of long–term debt funding.

5.3. Capital regulation

We now turn to the analysis of the effect of capital regulation on the ex–ante choice of the bank's optimal policies. Capital regulation in our model takes the form of restrictions on the (book) leverage ratio:

$$\frac{A + C^\pi(t) - (P_s \mathbb{1}_{\{t \le \tau^*\}} + P_l + P_d)}{A + C^\pi(t)} \ge k_{lev}.$$

The above constraint can be rewritten in terms of a level of liquid reserves at which the bank will be subject to mandatory liquidation:[24]

$$C^\pi(t) \ge \max\left\{ \frac{P_s \mathbb{1}_{\{t \le \tau^*\}} + P_l + P_d}{1 - k_{lev}} - A, 0 \right\} \equiv \underline{c}(k_{lev}). \tag{16}$$

Again, we solve the shareholders' problem numerically by taking into account regulatory Constraint (16). The results of our numerical analysis, reported in Table 4, show that tighter capital requirements induce the bank to target a lower leverage ratio by making cuts on *both* repos and long–term debt. This suggests that introducing special liquidity requirements to curb the banks' reliance on repos might be redundant, since the desired effect can be achieved via capital requirements alone.

Given the negative relation between the leverage ratio and the optimal target leverage, it would be interesting to explore whether the bank would cut more on the use of repos or on long–term debt. The ratio $P_s^*/(P_s^* + P_l^*)$, reported in Table 4, illustrates an apparent difference in the reaction of lowly–profitable and highly–profitable banks on higher leverage ratio. In particular, it suggests that, when faced with high capital requirements, low–profit banks would cut less on the use of repos than high–profit banks.

It is also worthwhile to notice that, when faced with a tighter leverage ratio, the bank will adjust its financing structure so as to avoid regulatory liquidation at a strictly positive level of liquid reserves, i.e. $\underline{c}(k_{lev}) = 0$ for any level of k_{lev}. Under the assumption that the deadweight costs of hoarding liquidity exceed the riskless rate, this result is very robust to changes in the parameters. In fact, any change in the level of debt that would raise the bank's liquidation threshold from zero to a strictly positive level, would induce the bank to increase its target level of liquid reserves by the same amount (see e.g., Hugonnier and Morellec (2014)). As long as the marginal cost of increasing liquid reserves exceeds the forgone tax benefits resulting from a reduction of debt, the bank prefers to reduce its levels of debt so as to avoid being liquidated

[24]Notice that, before the run, the bank faces a higher liquidation threshold.

Table 4: Impact of leverage regulation

	k_{lev}	P_s	P_l	$\frac{P_s}{P_s+P_l}$	$\underline{c}(k_{lev})$	b_1	b_0	r_l	MLR	BLR	V_1^*
$\mu = 20\%$	0	0.154	0.561	0.22	0	0.5174	0.487	0.0629	0.75	0.87	0.48
	0.05	0.069	0.401	0.15	0	0.5014	0.4904	0.0582	0.69	0.8	0.46
	0.10	0.049	0.291	0.14	0	0.4968	0.4892	0.0567	0.66	0.76	0.43
	0.15	0.035	0.175	0.17	0	0.4917	0.4865	0.0556	0.63	0.71	0.4
	0.20	0.025	0.055	0.31	0	0.4862	0.4826	0.0548	0.6	0.67	0.37
$\mu = 30\%$	0	0.1	1.929	0.05	0	0.5077	0.4851	0.0556	0.76	0.91	0.9
	0.05	0.045	1.66	0.03	0	0.4798	0.4721	0.0537	0.71	0.85	0.87
	0.10	0.03	1.48	0.02	0	0.4674	0.4627	0.053	0.67	0.8	0.84
	0.15	0.021	1.294	0.02	0	0.456	0.4529	0.0525	0.64	0.76	0.79
	0.20	0.014	1.106	0.01	0	0.445	0.4431	0.0521	0.61	0.72	0.75

Notes: This table illustrates the impact of leverage regulation on the bank's policies. Parameters value are: $\rho = 5\%$, $r_d = 4.5\%$, $r = 2.5\%$, $\lambda = 0.05$, $\sigma = 18\%$, $\theta = 35\%$, $P_d = 2$ and $\eta = 0.3$.

at a strictly positive level of liquid reserves. This is in fact the channel through which capital regulation curbs the bank's appetite for a higher leverage in our setting. In other words, the interplay between the liquidation and payout policies works as a transmission channel of the disciplining effect of the capital regulation.

6. Conclusions

We developed a continuous–time model that focuses on the liability structure of a modern bank. The main question we addressed is what drives the choices between repos, which carry with then rollover risk, and risky, long–term debt, which is a stable source of funding. We showed that, being inherent to the use of short–term debt, rollover risk imposes negative externalities on long term creditors, which gives rise to an additional component in long–term debt spreads that we have labeled the shadow cost of repo financing. This shadow cost prevents the bank from selling large amounts of repos. Furthermore, we found that there is a channel of interaction between liquidity management and capital structure, given that liquid reserves serve as a buffer against the losses caused by a run of the repo creditors. This feature played an important role in our analysis, since the bank's ability to build larger liquid reserves has direct implications on its choices of debt structure. Our numerical analysis showed that banks with higher returns on risky assets exhibit a lower proportion of repos in their financing structure. In contrast, banks with lower returns on risky assets cannot afford to hold substantial amounts of liquid reserves; hence, they face a higher shadow cost and, as a consequence, are more reliant on repos. A similar effect is generated by insured deposits: a large volume of insured deposit in the bank's financing structure reduces the bank's effective earnings and weakens its capacity to maintain high levels of liquid reserves; thus, inducing the bank to increase its reliance on repos. We found that this effect is more pronounced when insured deposits are senior to long–term debt.

We also examined the effect of regulation on the bank's ex–ante choice of financing structure. We considered three regulatory tools: liquidity regulation in the spirit of the Basel III Liquidity Coverage Ratio, payout restrictions and leverage ratio. All in all, we found that all of these tools are capable of curbing the bank's appetite for repos. Under liquidity regulation, which requires the bank to maintain a minimum level of liquid reserves as a certain proportion of its volume of repos, the bank substitutes short–term debt funding by long–term debt. Payout restrictions induce a similar substitution effect, as the bank operates with higher target levels of liquid reserves and, thus, faces a lower shadow cost. Leverage regulation, however, induces the bank to lower the volumes of both repos and long–term debt, which suggests that developing special liquidity regulation in order to reduce the banks' reliance on repos might be redundant.

It has to be said that our model's versatility, which allowed us to consider debt of different maturities and seniorities, together with insured deposits, has as a downside: a large part of our analysis has to be done numerically. Whenever possible, we provided the mathematical reasoning behind the model's features but, unfortunately, this could not be done in all situations. A clear avenue for future research would be development of a fully dynamic structure of the bank's balance sheet, which is only partially the case in the current work. This could be used to address, for instance, bailouts and refinancing.

AppendixA. From Value Functions to HJB Variational Inequalities

In this appendix we explain how the differential characterizations of the value functions U_0 and U_1, respectively introduced in Sections 3.1 and 3.2, are obtained. We assume these functions to be twice continuously differentiable. We also prove that the mappings $c \mapsto U_i(c)$, $i = 0, 1$ are concave. Since the methodologies share many common features, we work in the sequel with the generic function U_i and mention, whenever required, where is it that the cases $i = 0$ and $i = 1$ diverge. In order to simplify the exposition we use the following notation:

$$f_0 := r_d P_d + r_l P_l, \quad f_1 := r_d P_d + r_l P_l + \mathbb{1}_{\{t \le \tau^*\}} r_s P_s,$$

$$dC_0^L(t) := (1-\theta)(\mu - f_0)dt + (1-\theta)\sigma dW(t) - dL(t)$$

and

$$dC_1^L(t) := (1-\theta)(\mu - f_1)dt + (1-\theta)\sigma dW(t) - dL(t) - \mathbb{1}_{\{t \le \tau^*\}} P_s dN(t).$$

AppendixA.1. Concavity

We first look at the concavity of U_i. Consider the reserves levels $c_1, c_2 > \underline{c}$ and let L_1 and L_2 be two corresponding admissible payout strategies. Let $\lambda \in (0, 1)$ and define

$$\tilde{c} := \lambda c_1 + (1-\lambda)c_2 \quad \text{and} \quad \tilde{L} := \lambda L_1 + (1-\lambda)L_2.$$

Clearly $\tilde{L}$ is admissible. Since $dC_i^{\tilde{L}} = \lambda dC_i^{L_1} + (1-\lambda)dC_i^{L_2}$ and the conditional–expectation operator is linear, we have

$$U_i(\tilde{c}) \ge U_i^{\tilde{L}}(\tilde{c}) = \lambda U_i^{L_1}(c_1) + (1-\lambda)U_i^{L_2}(c_2).$$

By definition, for all $\epsilon > 0$ the strategy L_1 can be chosen such that $U_i^{L_1}(c_1) \ge U_i(c_1) - \epsilon/2$, and analogously for $U_i(c_2)$. In other words, the expression

$$U_i(\tilde{c}) \ge \lambda U_i(c_1) + (1-\lambda)U_i(c_2) - \epsilon$$

holds for any positive ϵ; thus, the mapping $c \mapsto U_i(c)$ is concave.

AppendixA.2. Complementarity conditions

Next we consider the condition $1 \le U_i'$. By definition, for any $h, y > \underline{c}$ there exists a strategy L_y such that $U_i^{L_y}(y) \ge U_i(y) - h^2$. Let $\underline{c} < h < c$ and construct a strategy L by setting $L(t) = h + L^{c-h}(t)$, Then

$$U_i(c) \ge U_i^L(c) = h + U_i^{L_{c-h}}(c-h) \ge h + U_i(c-h) - h^2,$$

which is equivalent to

$$\frac{U_i(c) - U_i(c-h)}{h} \ge 1 - h.$$

By the differentiability of U_i, we may let h go to zero and conclude that $U_i'(c) \ge 1$ for all $c \ge \underline{c}$.

AppendixA.3. Differential characterizations

Below we show that for $c > \underline{c}$ it holds that

$$\mathcal{L}_0 U_0(c) := \mathcal{L}U_0(c) - (1-\theta) r_s P_s U_0'(c) \leq 0 \quad \text{and} \quad \mathcal{L}_1 U_1(c) := \mathcal{L}U_1(c) - \lambda\big[U_1(c) - U_0\big((c-P_s)_+\big)\big] \leq 0.$$

To this end, we fix a payout strategy L with corresponding liquid–reserves process C_i^L ($C_i^L(0) = c$) and apply Itô's formula to $g_i(t,c) := e^{-\rho t} U_i(c)$:

$$\begin{aligned} e^{-\rho t} U_i\big(C_i^L(t)\big) = U_i(c) &+ \int_0^t e^{-\rho s}\big((1-\theta)(\mu - f_i) U_i'\big(C_i^L(s)\big) - \rho U_i\big(C_i^L(s)\big)\big) ds \\ &+ \frac{1}{2}\int_0^t e^{-\rho s} U_i''(C_s^L) d[C_i^L, C_i^L]^c(s) \\ &+ \int_0^t e^{-\rho s}(1-\theta)\sigma U_i'(C_i^L(s)) dW(s) - \int_0^t e^{-\rho s} U_i'(C_i^L(s)) dL(s) \\ &+ \sum_{s\in\Gamma_i} e^{-\rho s}\big[U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))(C_i^L(s_+) - C_i^L(s))\big], \end{aligned} \tag{A1}$$

where Γ_1 is the set of discontinuities of L and Γ_2 is the set of discontinuities of $\mathbb{1}_{\{t\leq\tau^*\}} P_s N$. Since L and $\mathbb{1}_{\{t\leq\tau^*\}} P_s N$ are of bounded variation, we have that

$$d[C_i^L, C_i^L]^c(s) = (1-\theta)^2 \sigma^2 ds.$$

Thus, Equation (A1) becomes

$$\begin{aligned} e^{-\rho t} U_i(C_i^L(t)) = U_i(c) &+ \int_0^t e^{-\rho s} \mathcal{L} U_i(C_i^L(s)) ds \\ &+ \int_0^t e^{-\rho s}(1-\theta)\sigma U_i'(C_i^L(s)) dW(s) - \int_0^t e^{-\rho s} U_i'(C_i^L(s)) dL(s) \\ &+ \sum_{s\in\Gamma} e^{-\rho s}\big(U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))(C_i^L(s_+) - C_i^L(s))\big), \end{aligned} \tag{A2}$$

If we take expectations on both sides of Equation (A2), the Itô integral vanishes and, using the Dynamic Programming Principle, we obtain

$$\mathbb{E}\left[e^{-\rho t} U_i(C_i^L(t))\right] \leq U_i(c) - \mathbb{E}\Big[\int_0^t e^{-\rho s} U_i'(C_i^L(s)) dL(s)\Big]. \tag{A3}$$

Notice that for the Poisson jump we have

$$\sum_{0\leq s\leq t} \mathbb{E}\left[e^{-\rho s}\big(U_1(C_1^L(s_+)) - U_1(C_1^L(s))\big)\right] = -\lambda \int_0^t e^{-\rho s}(U_1(C_1^L(s)) - U_0(C_1^L(s) - P_s)) ds. \tag{A4}$$

Expressions (A3) and (A4) yield

$$0 \geq \mathbb{E}\Big[\int_0^t e^{-\rho s} \mathcal{L}_i U_i(C_s^L) ds\Big] + \mathbb{E}\Big[\sum_{s\in\Gamma} e^{-\rho s}\big(U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))(C_i^L(s_+) - C_i^L(s))\big)\Big], \tag{A5}$$

where Γ is the set of discontinuities of L. By the Mean Value Theorem, there exists $\hat{c} \in (C_i^L(s_+), C_i^L(s))$ such that

$$U_i(C_i^L(s_+)) - U_i(C_i^L(s)) = U_i'(\hat{c})\big(C_i^L(s_+) - C_i^L(s)\big).$$

Therefore

$$U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))\big(C_i^L(s_+) - C_i^L(s)\big) = \big(U_i'(\hat{c}) - U_i'(C_i^L(s))\big)\big(C_i^L(s_+) - C_i^L(s)\big)$$

and, by concavity of U_i, the right–hand side of the above expression, as well as the second term on the right–hand side of Expression (A5) are positive. This yields

$$0 \geq \mathbb{E}\Big[\int_0^t e^{-\rho s}\mathcal{L}_i U_i(C_i^L(s))ds\Big]. \tag{A6}$$

Next we multiply both sides of the equation above times $1/t$. Since

$$\frac{1}{t}\int_0^t e^{-\rho s}\mathcal{L}_i U_i(C_s^L)ds \leq \max_{s\in[0,t]} e^{-\rho s}\big|\mathcal{L}_i U_i(C_s^L)\big| < \infty,$$

we may apply Lebesgue's Dominated Convergence Theorem and take the limit as $t \to 0$ inside the expectation operator, which yields

$$\mathcal{L}_0 U_0(c) \leq 0 \quad \text{and} \quad \mathcal{L}_i U_i(c) \leq 0.$$

AppendixA.4. Variational inequalities

So far, we have shown that U_i satisfies, for $c > \underline{c}$, the set of variational inequalities

$$\mathcal{L}_i U_i(c) \leq 0 \quad \text{and} \quad 1 - U_i'(c) \leq 0.$$

Our next task is to prove that one of the inequalities is always tight. In order to do so, we resort to the Dynamic Programming Principle and write, for $t > 0$,

$$U_i(c) = \max_{L\in\mathcal{A}} \mathbb{E}\Big[\int_0^t e^{-\rho s}dL_s + e^{-\rho t}U_i(C_i^L(t))\Big].$$

Inserting Equation (A2) into the equation above we obtain

$$\begin{aligned} 0 = \sup_{L\in\mathcal{A}} \Bigg\{ &\mathbb{E}\Big[\int_0^t e^{-\rho s}\mathcal{L}_i U_i(C_i^L(s))ds\Big] \\ &+ \mathbb{E}\Big[\int_0^t e^{-\rho s}(1 - U_i'(C_i^L(s)))dL(s)\Big] \\ &+ \mathbb{E}\Big[\sum_{s\in\Gamma} e^{-\rho s}\big(\Delta U_i(C_i^L(s)) - U_i'(C_i^L(s))\Delta C_i^L(s)\big)\Big]\Bigg\}. \end{aligned} \tag{A7}$$

If we write $\tilde{L}$ for the continuous pars of L, then Equation (A7) may be rewritten as

$$\begin{aligned} 0 = \sup_{\pi\in\Pi_p} \Bigg\{ &\mathbb{E}\Big[\int_0^t e^{-\rho s}\mathcal{L} U_i(C_i^L(s))ds\Big] \\ &+ \mathbb{E}\Big[\int_0^t e^{-\rho s}(1 - U_i'(C_i^L(s)))d\tilde{L}(s)\Big] \\ &+ \mathbb{E}\Big[\sum_{s\in\Gamma} e^{-\rho s}\big(\Delta U_i(C_i^L(s)) + \Delta L(s)\big)\Big]\Bigg\}. \end{aligned}$$

Notice that for all $s \in (0,t)\bigcap\Gamma$ it holds that

$$\Delta U_i(C_i^L(s)) + \Delta L(s) = \int_{C_i^L(s)-\Delta L(s)}^{C_i^L(s)} (1 - U_i'(c))dc \leq 0.$$

This implies all summands on the right–hand side of Equation (A7) are non positive; thus, for $c > \underline{c}$ it holds that

$$\max\left\{\mathcal{L}_i U_i(c), 1 - U_i'(c)\right\} = 0.$$

AppendixA.5. Suboptimality of liquidation at any level $\underline{c} > 0$

AppendixB. Dividend Barriers and Payout Strategies

In this appendix we provide verification theorems for the value functions U_0 and U_1 introduced in Sections 3.1 and 3.2, respectively. We proceed in the following way: On a first step, we assume there exist equilibrium r_l^*, b_0^* and b_1^* such that $U_0(0; b_0^*) = \tilde{u}_0$, $D_1(b_1^*) = P_l$ and $U_1(0; b_1^*) = 0$ and prove the results concerning optimality of the payout strategies and of $U_i(\cdot; b_i^*)$, for $i = 0, 1$. On a second step, we show that for $r_l, P_l \geq 0$ given, the equation

$$U_0(0; b_0^*(r_l, P_l)) = \tilde{u}_0$$

has a unique positive solution and that, provided that $b_1^* \in (P_s, P_s + b_0^*]$, there exists no other $\tilde{b}_1 \in (P_s, P_s + b_0^*]$ such that

$$\begin{aligned} \mathcal{L}_1 U_1(c) &= 0, \quad c \in (\underline{c}, \tilde{b}_1); \\ U_1(c) - U_1(\tilde{b}_1 -) + \tilde{b}_1 - c &= 0, \quad c \geq \tilde{b}_1, \end{aligned} \tag{A1}$$

together with the corresponding boundary conditions, is satisfied.

AppendixB.1. Verification theorems

Let $r_l^*, P_l, P_s \geq 0$ be such that $\mu - f_i > 0$ and assume there exists b_i^* such that $U_i(0; b_i^*) = 0$ (if $\tilde{u}_0 > 0$, the proof is analogous). Let the processes (C_i^*, L_i^*) be a solution to the following Skorokhod problem defined on $[\underline{c}, b_i^*]$:

$$C_i^*(t) = c + \int_0^t (\mu - f_i) ds + \int_0^t \sigma dW(s) - L_i^*(t); \tag{A2}$$

$$\text{for all } 0 \leq t \leq \tau_i^*, \ \underline{c} \leq C^*(t) \leq b_i^*; \tag{A3}$$

$$\int_0^{\tau_i^*} \mathbb{1}_{\{C_i^*(t) < b_i^*\}} dL_i^*(t) = 0, \tag{A4}$$

where $\tau_i^* := \inf\{t > 0 | C_i^*(t) \leq \underline{c}\}$. The solution to the so called Skorokhod Problem (A2)–(A4) can be found, for instance, in Karatzas and Shreve (1991). The process L_i^* is the *local time* of C_i^* at level b_i^*. Its effect on the dynamics of C_i^* is to reflect the latter downwards at level b_i^* in order to constrain it to $[\underline{c}, b_i^*]$. From Equation (A4) we see that the mass of the measure $dL_i^*(t)$ is carried by the set $\{C_i^*(t) = b_i^*\}$; thus, $L_i^*(t)$ is inactive whenever $C_i^*(t) < b_i^*$.

Now, let us show that $U_i(c; b_i^*) = \sup_L U_0^{(x_i, P_l, L)}(c)$, where $x_0 = 0$ and $x_1 = P_s$. Consider an admissible payoff strategy L_i and an initial level of liquid reserves $c_0 > \underline{c}$. Recall that the corresponding liquid–reserves process evolves according to the stochastic differential equation

$$dC_i^L(t) = (1 - \theta)\big((\mu - f_i)dt + \sigma dW(t)\big) - dL_i(t), \quad C_i^{L_i}(0) = c_0.$$

Proceeding as in Appendix AppendixA, we use the generalized Itô formula applied to $\tilde{g}_i(t, c) = e^{-\rho t} U_i(c; b_i^*)$. Using the fact that $\mathcal{L}_i U_i(C_i^{L_i}(t); b_i^*) \leq 0$, we obtain, after simplifications,

$$\begin{aligned} e^{-\rho t}\, \mathbb{E}\left[U_i(C_i^{L_i}(t); b_i^*)\right] \leq U_i(c; b_i^*) - \mathbb{E}\left[\int_0^t e^{-\rho s} U_i'(C_i^{L_i}(s); b_i^*)\big(d\tilde{L}_i(s)\big)\right] \\ + \mathbb{E}\left[\sum_{s \in \Gamma_i} e^{-\rho s}\big(U_i(C_i^{L_i}(s_+); b_i^*) - U_i(C_i^{L_i}(s); b_i^*)\big)\right], \end{aligned} \tag{A5}$$

where $\tilde{L}_i(s)$ is the continuous part of L_i. Let $s \in \Gamma_i$, then by the Mean Value Theorem and the fact that $U_i'(C_i^{L_i}(s); b_i^*) \geq 1$, there exists $\hat{c} \in (C_i^{L_i}(s_+), C_i^{L_i}(s))$ such that

$$U_i(C_i^{L_i}(s_+); b_i^*) - U_i(C_i^{L_i}(s); b_i^*) = U_i'(\hat{c}; b_i^*)\big(C_i^{L_i}(s_+) - C_i^{L_i}(s)\big) \leq L_i(s_+) - L_i(s) = -\Delta L_i(s).$$

Inserting the above expression into Expression (A5) we get

$$e^{-\rho t}\, \mathbb{E}\left[U_i(C_i^{L_i}(t); b_i^*)\right] \leq U_i(c; b_i^*) - \mathbb{E}\left[\int_0^t e^{-\rho s} dL_i(s)\right].$$

By continuity, $U_i(c; b_i^*)$ is bounded for $c \in [\underline{c}, b_i^*]$ and it grows linearly as c tends to infinity, therefore,

$$\lim_{t\to\infty} e^{-\rho t}\, \mathbb{E}\left[U_i(C_i^{L_i}(t); b_i^*)\right] = 0.$$

This implies that, if we set $dL_i(t) \equiv 0$ for all $t \geq \tau_i^*$,

$$U_i(c; b_i^*) \geq \mathbb{E}\left[\int_0^{\tau^*} e^{-\rho s} dL_i(s)\right]. \tag{A6}$$

Next, we consider the strategy L_i^*. Since the latter is the local time of C_i^* at level b_i^*, we may assume that $c \in [\underline{c}, b_i^*]$. Furthermore, L_i^* is a continuous processes, and on $(\underline{c}, b_i^*)$ it holds that $\mathcal{L}U_i(C_i^{L_i^*}(s); b_i^*) = 0$. Hence, for the strategy L_i^*, Itô's formula yields

$$\begin{aligned} e^{-\rho t} U_i(C_i^{L_i^*}(t); b_i^*) = U_i(c; b_i^*) + \int_0^t e^{-\rho s} \sigma U_i'(C_i^{L_i^*}(s); b_i^*) dW(s) \\ - \int_0^t e^{-\rho s} U_i'(C_i^{L_i^*}(s); b_i^*) dL_i^*(s). \end{aligned} \tag{A7}$$

The measure $dL_i^*(s)$ is supported on $\{C_i^{L_i^*}(s) = b_i^*\}$ and $U_i'(b_i^*) = 1$. Therefore, taking expectations, Equation (A7) may be rewritten as

$$e^{-\rho t}\, \mathbb{E}\left[U_i(C^{L_i^*}(t)_i; b_i^*)\right] = U_i(c; b_i^*) - \mathbb{E}\left[\int_0^t e^{-\rho s} dL_i^*(s)\right].$$

Letting t tend to ∞ we have

$$U_i(c; b_i^*) = \mathbb{E}\left[\int_0^{\tau_i^*} e^{-\rho s} dL_i^*(s)\right], \tag{A8}$$

which is equivalent to $U_i(c; b_i^*) = U^{(x_i, P_l, L^*)}(c)$. Form Equation (A6), we have that for any admissible L, it holds that $U_i(c; b_i^*) \geq U_i^{(x_i, P_l, L)}(c)$. Since L_i^* is admissible, Equation (A8) yields

$$U_i(c; b_i^*) = \sup_L U_0^{(x_i, P_l, L)}(c).$$

AppendixB.2. Uniqueness of b_0^* and b_1^*

Next we shown that b_0^* is the unique solution to $U_0(\underline{c}; b) = \max\{\eta A - P_d - P_l, 0\}$. Recall that $\beta_2 < 0 < \beta_1$ and observe that

$$U_0(\underline{c}; 0) = \frac{\beta_1 + \beta_2}{\beta_1\, \beta_2} = \frac{(1-\theta)(\mu - f_0)}{\rho} > 0. \tag{A9}$$

The mapping $b \mapsto U_0(\underline{c}; b)$ is decreasing, as shown by the condition

$$\frac{\partial U_0(\underline{c}; b)}{\partial b} = \frac{1}{\beta_1 - \beta_2}\left(\beta_2 e^{-\beta_1 b} - \beta_1 e^{-\beta_2 b}\right) < 0, \quad \text{for all } b > 0.$$

Furthermore, $\lim_{b\to\infty} U_0(\underline{c}; b) = -\infty$. Hence, the equation $U_0(\underline{c}; b) = 0$ has a unique solution. On the other hand, if $\eta A - P_d - P_l > 0$, then long–term debt is riskless and $r_l \equiv \rho$, and Equation (A9) becomes $U_0(\underline{c}; \underline{c}) = \rho^{-1}(1-\theta)(\mu - f_0)$. The relation

$$\eta A - P_d - P_l < \rho^{-1}(1-\theta)(\mu - f_0)$$

follows immediately from the fact that $A = (1-\theta)\mu/\rho$.

In order to show there is no other $\tilde{b}_1 \in (P_s, P_s + b_0^*]$ such that System (A1), together with the corresponding boundary conditions, is satisfied, we define a parametric family of functions $\left\{U_1(\cdot : b) \middle| b \in (P_s, P_s + b_0^*]\right\}$ as the solutions to

$$\begin{aligned} \mathcal{L}_1 U_1(c) &= 0, \quad c \in (\underline{c}, b); \\ U_1(c) - U_1(b-) + b - c &= 0, \quad c \geq b, \end{aligned}$$

together with the boundary conditions $U_1'(b; b) = 1$ and $U_1''(b; b) = 0$. By definition, $U_1(\underline{c}; b_1) = 0$, so we must show that $U_1(\underline{c}; b) \neq 0$ for all $b \neq b_1$. We will do this by showing that the mapping $b \mapsto U_1(\underline{c}; b)$ is strictly decreasing. We require the following auxiliary lemma:

Lemma 1. *Let m be a solution to*

$$\mathcal{L}m(c) - \lambda m(c) := \mathcal{L}_2 m(c) = 0, \quad \textit{for} \quad c \in (\underline{c}, b)$$

such that $m'(b) < 0$ and $m(b) > 0$. Then $m'(c) < 0$ for all $c \in (\underline{c}, b)$.

Proof. Assume that $m'(c) < 0$ does not hold for all $c \in (\underline{c}, b)$, and let $\bar{c}$ be the largest value on $(\underline{c}, b)$ that satisfies $m'(\bar{c}) = 0$. By construction, $\bar{c}$ is a positive local maximum of m. Since $m''(\bar{c}) \leq 0$, however, we would require $m(\bar{c}) \leq 0$ in order to have $\mathcal{L}_2 m(\bar{c}) = 0$. Therefore $m'(c) < 0$ for all $c \in (\underline{c}, b)$.

□

Next we consider two arbitrary payout thresholds b_1 and b_2 such that $b_1 < b_2$ and define

$$m(c) := U_1(c; b_1) - U_1(c; b_2), \quad c \in [c, b_1].$$

It is straightforward to show that m satisfies $\mathcal{L}_2 m(c) = 0$ subject to the boundary conditions $m'(b_1) = 1 - U_1'(b_1; b_2) < 0$ and $m''(b_1) = -U_1''(b_1; b_2) \geq 0$. Furthermore

$$\begin{aligned} m(b_1) &= U_1(b_1; b_1) - U_1(b_1; b_2) \\ &= U_1(b_1; b_1) - \left[U_1(b_2; b_2) - \int_{b_1}^{b_2} U_1'(c; b_2)\,\mathrm{d}c\right] \end{aligned} \tag{A10}$$

It follows from the Mean Value Theorem that there exists $\hat{b} \in [b_1, b_2]$ such that

$$\int_{b_1}^{b_2} U_1'(c; b_2)\,\mathrm{d}c = U_1'(\hat{b}; b_2) \cdot (b_2 - b_1) \geq (b_2 - b_1).$$

Therefore, given that $U_1(b_1, b_1) = U_1(b_2, b_2) = (\rho + \lambda)^{-1}(1-\theta)(\mu - r_l\, P_l - r P_s)$,

$$m(b_1) \geq U_1(b_1, b_1) - U_1(b_2, b_2) + (b_2 - b_1) > 0.$$

Lemma 1 then implies that $m'(c) < 0$ holds for all $c \in (\underline{c}, b)$. We conclude that $b \mapsto U_1(\underline{c}; b)$ is a strictly decreasing mapping.

AppendixB.3. Equality of c_0^* and b_1^*

Given a debt structure (P_s, P_l, P_d) and a choice c_0 of initial reserves level, the banks's ex–ante value is given by the expression

$$V_1(c_0, P_s, P_l) = U_1(c_0) - c_0 - (I - P_l - P_s - P_d).$$

Since U_1 is concave, the first–order conditions are sufficient to determine the optimal choice of c_0; that is, c_0^* is characterized by the equation

$$\frac{\partial}{\partial c_0} V_1(c_0, P_s, P_l) = U_1'(c_0) - 1 = 0. \tag{A11}$$

We know from Section AppendixB.2 that the unique admissible solution to Equation (A11) is precisely b_1^*.

AppendixC. Valuation of Contingent Claims

In this appendix we derive the ex–ante value of bank equity and debt. Recall that:

$$\mathcal{L}_1 g(c) = (1-\theta)^2 \frac{\sigma^2}{2} g''(c) + (1-\theta)(\mu - f_1) g'(c) - \rho g(c), \tag{C1}$$

where $f_1 = r_d P_d + r_l P_l + r_s P_s$ and g is a twice continuously–differentiable function.

AppendixC.1. Equity value

For a given debt structure (P_s, P_l, P_d), an interest rate on long–term debt r_l and a liquidation threshold $\underline{c}$, consider a payout barrier b_1 that satisfies $\underline{c} + P_s < b_1 \leq \underline{c} + P_s + b_0^*(P_l, r_l)$. If the run of repo creditors occurs in the region $c \in (\underline{c} + P_s, b_1]$, the bank survives and finds itself on the payout–retention region $(\underline{c}, b_0^*(P_l, r_l))$. In contrast, if the run occurs in the region $[\underline{c}, \underline{c} + P_s]$, the bank is liquidated and the shareholders receive

$$\tilde{u}_1(c) = \max\{\eta A + c - P_d - P_l - P_s, 0\}$$

upon the liquidation of the assets.

In principle, one must distinguish between three possible scenarios: In the first scenario, $\tilde{u}_1(\underline{c} + P_s) < 0$, so that shareholders receive nothing should a run–triggered liquidation occur. Another possibility is $\tilde{u}_1(\underline{c} + P_s) > 0$ but $\tilde{u}_1(\underline{c}) < 0$, which means that, should a run–triggered liquidation occur, shareholders may receive a positive amount if the current level of liquid reserves is sufficiently high. The last possibility is having $\tilde{u}_1(\underline{c}) > 0$, which means that long–term debt is riskless and shareholders will always collect a strictly positive amount in the event of liquidation. We present below the design of equity value for the case $\tilde{u}_1(\underline{c} + P_s) < 0$, as it is the only case that manifests itself in our numerical analysis.

Recall that U_0 denotes the post–run optimal equity value defined in Section 3.1. Before a run occurs, the equity value satisfies the system

$$\mathcal{L}_1 U_1(c) - \lambda U_1(c) = 0, \quad c \in (\underline{c}, \underline{c} + P_s), \tag{C2}$$

$$\mathcal{L}_1 U_1(c) - \lambda [U_1(c) - U_0(c - P_S)] = 0, \quad c \in (\underline{c} + P_s, b_1), \tag{C3}$$

$$U_1(c) - U_1(b_1) + b_1 - c = 0, \quad c \geq b_1, \tag{C4}$$

subject to the following conditions:

$$\begin{aligned} U_1''(b_1) &= 0, \\ U_1'(b_1) &= 1, \\ \lim_{c \uparrow \underline{c}+P_s} U_1(c) &= \lim_{c \downarrow \underline{c}+P_s} U_1(c), \\ \lim_{c \uparrow \underline{c}+P_s} U_1'(c) &= \lim_{c \downarrow \underline{c}+P_s} U_1'(c). \end{aligned}$$

Applying the Method of Variation of Parameters, we can show that a particular solution to Equation (C3) is given by

$$H(c) = \kappa_1 e^{\beta_1 c} + \kappa_2 e^{\beta_2 c},$$

where

$$\kappa_1 = \frac{\beta_2}{\beta_1} \frac{\kappa_0}{(\beta_1 - \gamma_1)(\beta_1 - \gamma_2)} e^{-\beta_1 (P_s + b_0^*)} \quad \text{and} \quad \kappa_2 = -\frac{\beta_1}{\beta_2} \frac{\kappa_0}{(\beta_2 - \gamma_1)(\beta_2 - \gamma_2)} e^{-\beta_2 (P_s + b_0^*)},$$

with

$$\kappa_0 = \frac{1}{(1-\theta)^2} \frac{2}{\sigma^2} \frac{\lambda}{(\beta_1 - \beta_2)},$$

and $\gamma_1 > 0$ and $\gamma_2 < 0$ are the roots of the characteristic polynomial

$$(1-\theta)^2 \frac{\sigma^2}{2} \gamma^2 + (1-\theta)(\mu - f_1)\gamma = \rho + \lambda.$$

Then, the equity value function can be defined as follows:

$$U_1(c) = \begin{cases} A_{21}(b_1) e^{\gamma_1 c} + A_{22}(b_1) e^{\gamma_2 c}, & c \in (\underline{c}, \underline{c} + P_s); \\ A_{11}(b_1) e^{\gamma_1 c} + A_{12}(b_1) e^{\gamma_2 c} + \kappa_1 e^{\beta_1 c} + \kappa_2 e^{\beta_2 c}, & c \in (\underline{c} + P_s, b_1); \\ \frac{(1-\theta)(\mu - f_1)}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda} U_0^*(b_1 - P_s) + c - b_1, & c \geq b_1, \end{cases} \tag{C5}$$

where

$$\begin{aligned} A_{11}(b_1) &= -\left[\frac{\gamma_2(1 - H'(b_1)) + H''(b_1)}{\gamma_1(\gamma_1 - \gamma_2)}\right] e^{-\gamma_1 b_1}, \\ A_{12}(b_1) &= \left[\frac{\gamma_1(1 - H'(b_1)) + H''(b_1)}{\gamma_2(\gamma_1 - \gamma_2)}\right] e^{-\gamma_2 b_1}, \\ A_{21}(b_1) &= A_{11}(b_1) - \left[\frac{\gamma_2 H(\underline{c} + P_s) - H'(\underline{c} + P_s)}{(\gamma_1 - \gamma_2)}\right] e^{-\gamma_1(\underline{c} + P_s)} \quad \text{and} \\ A_{22}(b_1) &= A_{12}(b_1) + \left[\frac{\gamma_1 H(\underline{c} + P_s) - H'(\underline{c} + P_s)}{(\gamma_1 - \gamma_2)}\right] e^{-\gamma_2(\underline{c} + P_s)}. \end{aligned}$$

AppendixC.2. The value of long–term debt

Here we derive the market value of long–term, risky debt. We assume that $\underline{c} + P_s < b_1 < \underline{c} + P_s + b_0^*$. The market value of risky, long–term debt satisfies the following system:

$$\mathcal{L}_1 D_1(c) + r_l P_l - \lambda[D_1(c) - D_0^*(c - P_s)] = 0, \quad c \in (\underline{c} + P_s, b_1); \tag{C6}$$

$$\mathcal{L}_1 D_1(c) + r_l P_l - \lambda[D_1(c) - \tilde{d}_1(c)] = 0, \quad c \in (\underline{c}, \underline{c} + P_s), \tag{C7}$$

where D_0^* denotes the post–run value of long–term debt defined in Section 3.1, together with the following boundary and pasting conditions:

$$\begin{aligned} D_1'(b_1) &= 0, \\ \lim_{c\uparrow \underline{c}+P_s} D_1(c) &= \lim_{c\downarrow \underline{c}+P_s} D_1(c), \\ \lim_{c\uparrow \underline{c}+P_s} D_1'(c) &= \lim_{c\downarrow \underline{c}+P_s} D_1'(c), \\ D_1(\underline{c}) &= \tilde{d}_1(\underline{c}). \end{aligned}$$

Let M_1 denote a particular solution to the non–homogeneous Equation (C6):

$$M_1(c) = \chi_0(\chi_{11}e^{\beta_1 c} - \chi_{12}e^{\beta_2 c}) - \chi_{13},$$

where

$$\chi_0 = \frac{\lambda}{(1-\theta)^2}\frac{2}{\sigma^2}\left(\frac{\tilde{d}_0 - \frac{r_l P_l}{\rho}}{\beta_1 e^{\beta_1 b} - \beta_2 e^{\beta_2 b + (\beta_1-\beta_2)\underline{c}}}\right),$$

$$\chi_{11} = \frac{\beta_2}{(\beta_1-\gamma_1)(\beta_1-\gamma_2)}e^{\beta_2(b-\underline{c})-\beta_1 P_s}, \quad \chi_{12} = \frac{[\beta_1 e^{\beta_1 b} - \beta_2 e^{\beta_2 b}(1-e^{(\beta_1-\beta_2)\underline{c}})]}{(\beta_2-\gamma_1)(\beta_2-\gamma_2)}e^{-\beta_2(\underline{c}+P_s)},$$

$$\chi_{13} = \frac{1}{(1-\theta)^2}\frac{2}{\sigma^2}\frac{r_l P_l}{\gamma_1\gamma_2}\frac{(\rho+\lambda)}{\rho}.$$

Let $M_2(c)$ denote a particular solution to the non–homogeneous Equation (C7). The general solution of System (C6)-(C7) is

$$D_1(c) = \begin{cases} B_{21}(b_1)e^{\gamma_1 c} + B_{22}(b_1)e^{\gamma_2 c} + M_2(c), & c \in [\underline{c}, \underline{c}+P_s]; \\ B_{11}(b_1)e^{\gamma_1 c} + B_{12}(b_1)e^{\gamma_2 c} + M_1(c), & c \in (\underline{c}+P_s, b_1]. \end{cases} \tag{C8}$$

In order to precise the coefficients $B_{11}(b_1), B_{12}(b_1), B_{21}(b_1)$ and $B_{22}(b_1)$, we introduce the following auxiliary functions:

$$\begin{aligned} G_1(b_1) &= (\tilde{d}_1(\underline{c}) - M_2(\underline{c}))e^{\gamma_1 P_s} + M_2(P_s+\underline{c}) - M_1(P_s+\underline{c}) + \frac{M_1'(b_1)}{\gamma_1}e^{\gamma_1(P_s-b_1)+\gamma_1\underline{c}}, \\ G_2(b_1) &= \gamma_1(\tilde{d}_1(\underline{c}) - M_2(\underline{c}))e^{\gamma_1 P_s} + M_2'(P_s+\underline{c}) - M_1'(P_s+\underline{c}) + M_1'(b_1)e^{\gamma_1(P_s-b_1)+\gamma_1\underline{c}}, \\ F_1(b_1) &= e^{\gamma_2(P_s+\underline{c})} - \frac{\gamma_2}{\gamma_1}e^{\gamma_2 b_1+\gamma_1(P_s-b_1)+\gamma_1\underline{c}}, \\ F_2(b_1) &= \gamma_2[e^{\gamma_2(P_s+\underline{c})} - e^{\gamma_2 b_1+\gamma_1(P_s-b_1)+\gamma_1\underline{c}}], \\ g_1 &= e^{\gamma_1 P_s+\gamma_2\underline{c}} - e^{\gamma_2 P_s+\gamma_2\underline{c}} \quad \text{and} \\ g_2 &= \gamma_1 e^{\gamma_1 P_s+\gamma_2\underline{c}} - \gamma_2 e^{\gamma_2 P_s+\gamma_2\underline{c}}. \end{aligned}$$

Solving the above system of boundary, value–matching and smooth–pasting conditions yields:

$$\begin{aligned} B_{12}(b_1) &= \frac{G_2(b_1)g_1 - G_1(b_1)g_2}{F_2(b_1)g_1 - F_1(b_1)g_2}, \\ B_{11}(b_1) &= -\left[\frac{M_1'(b_1) + \gamma_2 B_{12}(b_1)e^{\gamma_2 b_1}}{\gamma_1}\right]e^{-\gamma_1 b_1}, \\ B_{21}(b_1) &= [\tilde{d}_1(\underline{c}) - M_2(\underline{c}) - B_{22}(b_1)e^{\gamma_2\underline{c}}]e^{-\gamma_1\underline{c}} \quad \text{and} \\ B_{22}(b_1) &= \frac{G_1(b_1) - B_{12}(b_1)F_1(b_1)}{g_1}. \end{aligned}$$

The value of long–term debt when it is *senior* to deposits can be computed by using the above formulas with $\tilde{d}_1(c) = c + \eta A - P_s < P_l$ and $\tilde{d}_0 = \eta A$. In this case, the particular solution to Equation (C7) is given by

$$M_2(c) = \chi_{21} c + \chi_{22},$$

where

$$\chi_{21} = \frac{\lambda}{\rho + \lambda}, \qquad \chi_{22} = \frac{r_L P_l + \lambda(\eta A - P_s)}{\rho + \lambda} + (1-\theta)\frac{\lambda(\mu - f_1)}{(\rho+\lambda)^2}.$$

Consider now the value of long–term debt when it is *junior* to deposits. Assume that $P_d \geq \underline{c} + \eta A$, which implies that long–term creditors will receive nothing in the event of liquiation, i.e. $\tilde{d}_1(c) = 0$ and $\tilde{d}_0 = 0$. The market value of junior long–term debt satisfies the above formulas with the following modifications: $M_2(c) \equiv \frac{r_l P_l}{\rho+\lambda}$, $\tilde{d}_0 = 0$ and $\tilde{d}_1(c) = 0$.

AppendixD. Computing the Liquidation Probabilities

In this section we describe the numerical approximation of the liquidation probabilities discussed in Section 5.1. We require the following function spaces: $\mathbb{L}^2(D)$, the space of square–integrable functions over some open, bounded domain D; $\mathcal{C}^1\big((0,T); L^2(D)\big)$, the space of (weakly)–differentiable functions over $(0,T)$ with values in $\mathbb{L}^2(D)$, and $\mathbb{H}^1(D)$, the Sobolev space of square–integrable functions with square–integrable weak derivatives over D. For a thorough introduction to Sobolev spaces we refer the reader to Chapter 5 in Evans (1998).

We follow a methodology that is in line with most of the analysis in this paper and first approximate the liquidation probabilities after a run on repos has occurred. The numerical procedure can then be easily extended to the setting in which repos are still present in the bank's balance sheet.

AppendixD.1. Computing the probability of survival after the run

Technically, it is slightly simpler to deal with the probabilities of survival, from which the liquidation probabilities follow immediately. In other words, for a given strategy $\pi = (P_s, P_l, L)$ and a time horizon T, we study the object

$$H(t,c,T) = \mathbb{P}\{\tau_\pi > T \,|\, C^\pi(t) = c\},$$

i.e. the probability that the bank will not be liquidated before date T, given that its liquidity at date $t < T$ equals c.

In order to obtain a partial differential equation that describes H, we mimic the way in which we proceeded in AppendixA, noting that the presence of a time dimension results in a partial derivative with respect to time when making use of the Itô formula. Let $D := (\underline{c}, b_0)$, then for given time horizon T (taking advantage of the stationary nature of the liquid–reserves dynamics it is without loss of generality to consider the initial *evaluation date* to be $t = 0$), the mapping $(t,c) \mapsto H(t,c,T)$ solves the following boundary–value problem[25]:

$$\begin{aligned} \frac{\partial H(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2\frac{\partial^2 H(t,c)}{\partial c^2} + (1-\theta)(\mu - f_0)\frac{\partial H(t,c)}{\partial c} &= 0, \\ H(T,c) &= 1 \text{ for all } c > \underline{c}, \\ H(t,\underline{c}) &= 0, \\ \frac{\partial H}{\partial c}(t, b_0) &= 0. \end{aligned} \tag{D1}$$

Observe that there is no term $\rho\partial H(t,c)$ on the left–hand side of the differential equation. This obeys the fact that the probability of default is not discounted. The Dirichlet condition $H(t,\underline{c}) = 0$, which simply

[25]From this point on we drop the parameter T so as to simplify the notation

states that the probability of survival contingent on having been liquidated is zero, is precisely the reason why we work with the survival probabilities. We point out below why is it that having this zero boundary condition simplifies the problem at hand. The Neumann condition along the boundary $c \equiv b_0$ corresponds to the reflection of C^π at the said boundary. Namely, the probability of survival cannot be increasing at $c \equiv b_0$, since the liquid reserves are not permitted to increase beyond this level.

Next, we consider Problem (D1) in weak formulation: find a function

$$H \in \mathcal{C}\big((0,T);\mathbb{H}^1(D)\big) \cap \mathcal{C}^1\big((0,T);\mathbb{L}^2(D)\big)$$

such that for $t \in (t_0, T)$ and for all $\varphi \in H^1(D)$ (commonly referred to as *test functions*) the following holds:

$$\begin{aligned} \frac{d}{dt}\int_D H(t,c)\varphi(c)\,dc + \frac{1}{2}(1-\theta)^2\sigma^2 \int_D \frac{\partial^2 H(t,c)}{\partial c^2}\varphi(c)\,dc +& \\ (1-\theta)(\mu - f_0)\int_D \frac{\partial H(t,c)}{\partial c}\varphi(c)\,dc\,dc &= 0, \\ H(T,c) &= 1 \text{ for all } c > \underline{c}, \\ H(t,\underline{c}) &= 0, \\ \frac{\partial H}{\partial c}(t,b_0) &= 0. \end{aligned} \tag{D2}$$

On a further step, we change to time–to–maturity so as to have an *initial–value problem*, i.e. the time variable becomes $\mathbf{t} := T - t$, and we integrate the second–order term by parts. This results in the ensuing expression:

$$\begin{aligned} \frac{d}{d\mathbf{t}}\int_D H(\mathbf{t},c)\varphi(c)\,dc + \frac{1}{2}(1-\theta)^2\sigma^2 \int_D \frac{\partial H(\mathbf{t},c)}{\partial c}\frac{d\varphi(c)}{dc}\,dc -& \\ (1-\theta)(\mu - f_0)\int_D \frac{\partial H(\mathbf{t},c)}{\partial c}\varphi(c)\,dc &= \frac{(1-\theta)^2\sigma^2}{2}\int_{\partial D} \frac{\partial H(\mathbf{t},c)}{\partial c}\varphi(c)\,dc, \\ H(0,c) &= 1 \text{ for all } c > \underline{c}, \\ H(\mathbf{t},\underline{c}) &= 0, \\ \frac{\partial H}{\partial c}(\mathbf{t},b_0) &= 0. \end{aligned} \tag{D3}$$

We employ a finite element method in the spacial domain D and a θ–scheme (see e.g., Wilmott (2006) for a thorough presentation of θ–schemes) in time to solve Problem (D3). The idea behind the finite element method is to choose a finite–dimensional subspace $P_N \subset \mathbb{H}^1(D)$, for $N \in \mathbb{N}$, that incorporates the Dirichlet boundary condition, i.e. $\varphi_N(\underline{c}) = 0$, and to solve the approximated weak formulation. In other words, the problem becomes to find $H_N \in \mathcal{C}^1((0,T);P_N)$ such that for $\mathbf{t} \in (0,T)$

$$\begin{aligned} \frac{d}{d\mathbf{t}}\int_D H_N(t,c)\varphi_N(c)\,dc + \frac{1}{2}(1-\theta)^2\sigma^2 \int_D \frac{\partial H_N(\mathbf{t},c)}{\partial c}\frac{\partial\varphi_N(c)}{\partial c}\,dc -& \\ (1-\theta)(\mu - f_0)\int_D \frac{\partial H_N(\mathbf{t},c)}{\partial c}\varphi_N(c)\,dc &= 0 \quad \text{for all } \varphi_N \in P_N, \\ H_N(0,c) &= 1, \quad c > \underline{c}. \end{aligned} \tag{D4}$$

Observe that the term $\frac{1}{2}(1-\theta)^2\sigma^2\int_{\partial D}\frac{\partial H(\mathbf{t},c)}{\partial c}\varphi(c)\,dc$ has disappeared, since $\frac{\partial H(\mathbf{t},c)}{\partial c}\Big|_{c=b_0} = \varphi(\underline{c}) = 0$. If we were working with the liquidation probability instead of the survival one, then $\varphi(\underline{c})$ would not equal 0. This would result in a non–zero right–hand side in Equation (D4), which is slightly more cumbersome to deal with. By definition, the finite–dimensional subspace P_N admits a basis $(\mathbf{b}_j, j = 1, \ldots, N)$. In the latter, H_N and φ_N can be written as

$$H_N(\mathbf{t}, c) = \sum_{j=1}^{N} H_{N,j}(\mathbf{t})\mathbf{b}_j(c) \text{ and } \varphi_N(c) = \sum_{j=1}^{N} \varphi_{N,j}\mathbf{b}_j(c), \tag{D5}$$

where $(H_{N,j}, j = 1, \ldots, N)$ are the time–dependent coefficients of H_N. We have chosen to use the so–called *hat–function* basis, which is constructed as follows: the domain $(\underline{c}, b_0)$ is partitioned in N equally sized subintervals using the lattice $c_0 = \underline{c}, c_1, \ldots, c_N = b_0$. For any $1 < j \leq N - 1$, we define $\mathbf{b}_j$ so as to satisfy $\mathbf{b}_j(c_{j-1}) = 0, \mathbf{b}_j(c_j) = 1, \mathbf{b}_j(c_{j+1}) = 0$. Observe that this incorporates the boundary condition $\mathbf{b}_1(\underline{c}) = 0$. Finally $\mathbf{b}_N$ satisfies $\mathbf{b}_N(c_{N-1}) = 0$ and $\mathbf{b}_N(b_0) = 1$.

Inserting Expressions (D5) into Equation (D4) yields the semi–discrete formulation in matrix form: find $\underline{\mathrm{H}}_N \in C^1((0, T); \mathbb{R}^N)$ such that for $\mathbf{t} \in (0, T)$

$$\begin{aligned} \mathbf{M}\frac{d}{d\mathbf{t}}\,\underline{\mathrm{H}}_N(\mathbf{t}) + \mathbf{A}\,\underline{\mathrm{H}}_N(\mathbf{t}) &= 0, \\ \underline{\mathrm{H}}_N(0) &= \mathbf{1}, \end{aligned} \tag{D6}$$

where $\mathbf{1}$ is the N–dimensional vector of ones and $\underline{\mathrm{H}}_N = \left(H_{N,1}, \ldots, H_{N,N}\right)^\top$ is the *coefficients vector*. The matrices $\mathbf{A}, \mathbf{M} \in \mathbb{R}^{N\times N}$, which are commonly referred to as the *stiffness* and the *mass* matrix, respectively, are given by

$$\mathbf{M}_{ij} = \int_D \mathbf{b}_j(c)\mathbf{b}_i(c)\,dc$$

and

$$\mathbf{A}_{ij} = \frac{1}{2}(1-\theta)^2\sigma^2 \int_D \frac{\partial \mathbf{b}_j(c)}{\partial c}\frac{\partial \mathbf{b}_i(c)}{\partial c}\,dc - (1-\theta)(\mu - f_0)\int_D \frac{\partial \mathbf{b}_j(c)}{\partial c}\mathbf{b}_i(c)\,dc.$$

In order to have a fully discrete problem, we employ finite differences to approximate the time derivatives. To this end, for $m \in \mathbb{N}$, let us denote by $\mathcal{T} = \{\mathbf{t}_0, \ldots, \mathbf{t}_m\}$ a partition of the time interval $[0, T]$, with $\mathbf{t}_0 = 0$, $\mathbf{t}_m = T$ and $\mathbf{t}_i = i\,\Delta\mathbf{t}$, $\Delta\mathbf{t} = T/m$, $T > 0$. The θ–scheme, for $\theta \in [0, 1]$, is then given by

$$\mathbf{M}\,\frac{\underline{\mathrm{H}}_N(\mathbf{t}_{i+1}) - \underline{\mathrm{H}}_N(\mathbf{t}_i)}{\Delta\mathbf{t}} + \mathbf{A}\left(\theta\underline{\mathrm{H}}_N(\mathbf{t}_{i+1}) + (1-\theta)\underline{\mathrm{H}}_N(\mathbf{t}_i)\right) = 0.$$

When $\theta = 0$ we have the standard, fully–explicit finite–differences method. This converges rapidly, but might exhibit instabilities. On the opposite side of the spectrum, setting $\theta = 1$ yields a fully–implicit method that is unconditionally stable, but which converges slowly. Choosing $\theta = 0.5$ yields the so–called *Crank–Nicholson* scheme, which strikes a nice balance between stability and speed of convergence. The full approximate formulation of our problem in matrix form is as follows: for $i = 0, \ldots, m-1$ one must find a vector $\underline{\mathrm{H}}_N(\mathbf{t}_i)$ that satisfies

$$\begin{aligned} (\mathbf{M} + \Delta\mathbf{t}\,\theta\mathbf{A})\,\underline{\mathrm{H}}_N(\mathbf{t}_{i+1}) = (\mathbf{M} - \Delta\mathbf{t}\,(1-\theta)\mathbf{A})\,\underline{\mathrm{H}}_N(\mathbf{t}_i), \\ \underline{\mathrm{H}}_N(0) = \mathbf{1}. \end{aligned} \tag{D7}$$

Solving Problem (D7) yields, for each $\mathbf{t}_i$, the coefficient vector $\underline{\mathrm{H}}_N(\mathbf{t}_i)$, for $i = 1, \ldots, m$. This results in the following approximation of H:

$$\hat{H}_N(\mathbf{t}_i, c) := \sum_{j=1}^{N} H_{N,j}(\mathbf{t}_i)\mathbf{b}_j(c).$$

AppendixD.2. Computing the probability of survival before the run

Two additions must be made to the method described above so as to compute the survival probabilities before the run on repos takes place, which we shall denote $K(t, c)$. First, the possible jump of the liquid reserves as a result of a run has to be accounted for, which results in the following boundary–value problem:

$$\begin{aligned}\frac{\partial K(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2\frac{\partial^2 K(t,c)}{\partial c^2} + (1-\theta)(\mu - f_1)\frac{\partial K(t,c)}{\partial c} &= \\ \lambda[K(t,c) - H(t, c-P_s)_+]&, \\ K(T,c) &= 1 \text{ for all } c > 0, \\ K(t,\underline{c}) &= 0, \\ \frac{\partial K}{\partial c}(t, b_1) &= 0.\end{aligned} \tag{D8}$$

After rearranging we have that the partial differential equation in Expression (D8) can be written as

$$\frac{\partial K(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2\frac{\partial^2 K(t,c)}{\partial c^2} + (1-\theta)(\mu - f_1)\frac{\partial K(t,c)}{\partial c} - \lambda K(t,c) = 0 \tag{D9}$$

if $c \leq P_s + \underline{c}$, whereas in the case $P_s + \underline{c} < c < b_1$ we have

$$\frac{\partial K(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2\frac{\partial^2 K(t,c)}{\partial c^2} + (1-\theta)(\mu - f_1)\frac{\partial K(t,c)}{\partial c} - \lambda K(t,c) = -\lambda H(t, c - P_s). \tag{D10}$$

The discretization of the boundary–value problem in Expression (D8) can be done separately on the domains $(0,T) \times (\underline{c}, P_s + \underline{c})$ and $(0,T) \times (P_s + \underline{c}, b_1)$, respectively. In the former case we can do exactly as we did in Section AppendixD.1, with the substitution of λ for ρ. The latter case involves the second modification to the algorithm described above; namely, we now have a differential equation with a right–hand side. The equivalent to Equation (D4) is, for all $\varphi_M \in P_M$,

$$\begin{aligned}\frac{d}{d\mathbf{t}}\int_D K_M(\mathbf{t},c)\varphi_M(c)\,dc + \frac{(1-\theta)^2\sigma^2}{2}\int_D \frac{\partial K_M(\mathbf{t},c)}{\partial c}\frac{\partial \varphi_M(c)}{\partial c}\,dc + \lambda\int_D K_M(\mathbf{t},c)\varphi_M(c)\,dc - \\ (1-\theta)(\mu - f_0)\int_D \frac{\partial K_M(\mathbf{t},c)}{\partial c}\varphi_M(c)\,dc = \lambda\int_D H_M(\mathbf{t}, c - P_s)\varphi_M(c)\,dc, \\ K_M(0,c) = 1, \quad c \geq P_s + \underline{c},\end{aligned}$$

where M and N are not necessarily equal. Notice that $c - P_s \in [\underline{c}, b_0]$, but the discretizations in space need not match between cases. In order to address this, for a given discretization $(\tilde{c}_0 = \underline{c}, \tilde{c}_1, \ldots, \tilde{c}_M = b_1)$ we define $\tilde{c}_p$ to be the first element that is greater than or equal to $P_s + \underline{c}$. We then interpolate the values of H obtained on the mesh $\mathcal{T} \times (c_0, \ldots, c_N)$ to have the corresponding ones over $\mathcal{T} \times (\tilde{c}_0, \ldots, \tilde{c}_M)$. The advantage of using a hat–functions basis will now become clear, since the decomposition of the mapping $(t,c) \mapsto H_M(t, c - P_s)$ in the said basis $(\mathbf{b}_j, j = 1, \ldots, M)$ is

$$H_M(\mathbf{t}, c) = \sum_{j=1}^{N} H_M(\mathbf{t}, \tilde{c}_j)\mathbf{b}_j(c).$$

In other words, computing the coefficients in the basis decomposition is trivial, as they match the values of H_M on the nodes of the discretized time–space domain. In matrix form we have, for $j \geq p$, the system

$$\begin{aligned}\mathbf{M}\frac{d}{d\mathbf{t}}\,\underline{\mathrm{K}}_M(\mathbf{t}) + \mathbf{A}\,\underline{\mathrm{K}}_M(\mathbf{t}) &= \lambda\mathbf{M}\,\underline{\mathrm{H}}_M(t), \\ \underline{\mathrm{H}}_N(0) &= \mathbf{1},\end{aligned}$$

where $\underline{\mathrm{H}}_{M,j}(\mathbf{t})$ is the interpolate $H(\mathbf{t}, \tilde{c}_j - P_s)$. We can now proceed as in Section AppendixD.1 by setting $\underline{\mathrm{H}}_{M,j}(\mathbf{t}) = 0$ for all $j < p$. The ex–ante probability of liquidation is simply $P(T,c) = 1 - K(t,c)$.

AppendixE. Analyzing the Impact of Payout Restrictions on Financing Decisions

In this section we analyze the effect of a regulatory–constrained payout policy on the ex–ante value of equity that was introduced in Section 5.2. For simplicity we assume that $\underline{c} = 0$ and that $\tilde{u}_0(0) = \tilde{u}_1(P_s) = 0$. The case with general values for $\underline{c}$ and $U_i(\underline{c})$, $i = 0, 1$ is analogous, but the Arithmetic is more involved. Once more we first look at the impact of payout restrictions after the run on repos and analyze the general case afterwards.

AppendixE.1. The impact of payout restrictions after the run

Recall that, in the unregulated case, the equity value function U_0 satisfies the ordinary differential equation

$$\rho U_0(c) = (1-\theta)^2 \frac{\sigma^2}{2} U_0''(c) + (1-\theta)(\mu - f_0) U_0'(c) \tag{E1}$$

on the region $(\underline{c}, b_0^*)$. For each choice of r_l and P_l, Equation (E1) has the general solution

$$A(r_l, P_l) e^{\beta_1 c} + B(r_l, P_l) e^{\beta_2 c}.$$

Furthermore, under the assumptions $\underline{c} = 0$ and $U_0(0) = 0$ we have that $A(r_l, P_l) = -B(r_l, P_l)$. Notice that the choice of $A(r_l, P_l)$ determines the level of liquid reserves at which the condition $U_0' = 1$ is satisfied, but it bears no weight on where the second–order condition $U_0'' = 0$ is fulfilled. This occurs whenever

$$\beta_1^2 e^{\beta_1 c} - \beta_2^2 e^{\beta_2 c} = 0,$$

an equation whose solution is our old acquaintance

$$b_0^* = \frac{1}{\beta_2 - \beta_1} \log \left(\frac{\beta_1}{\beta_2} \right)^2.$$

In other words, b_0^* is the unique inflection point of the family of functions that satisfy Equation (E1) and the boundary condition at zero. The fact that we impose the condition $U_0' = 1$ precisely at $c = b_0^*$ is what allows for a C^2–linear continuation of U_0 over (b_0^*, ∞).

Assume now that the payout constraint $b_{reg} > b_0^*$ is imposed, i.e. dividends can only be distributed at date t if the level of liquid reserves $C^\pi(t) \geq b_{reg}$. In terms of the corresponding equity value function U_{reg}, this results in the Neumann boundary condition $U_{reg}'(b_{reg}) = 1$, which follows from the same argument as in the unregulated case and yields

$$U_{reg}(c) = \frac{1}{\beta_1 e^{\beta_1 b_{reg}} - \beta_2 e^{\beta_2 b_{reg}}} \left(e^{\beta_1 c} - e^{\beta_2 c} \right).$$

From the above argument we have that at $b_0^* < b_{reg}$ the mapping $c \mapsto U_{reg}(c)$ changes concavity. Furthermore, since $\beta_1 < 0 < \beta_2$, we have that $\lim_{c\to\infty} U_{reg}'(c) = \infty$. This fact, together with the exponential structure of U_{reg}, implies that U_{reg} is convex on (b_0^*, ∞). Analogously, U_{reg} is concave on $(-\infty, b_0^*)$, thus there exists a critical level of liquid reserves $c_0 < b_0^*$ such that $U_{reg}'(c_0) = 1$.

AppendixE.2. The impact of payout restrictions before the run

Under the assumptions that $\underline{c} = 0$ and $\tilde{u}_1(P_s) = 0$, the pre–run equity value function U_1 satisfies the following system:

$$\begin{aligned}
\mathcal{L}U_1(c) - \lambda U_1(c) &= 0, \quad c \in (0, P_s), && \text{(E2)}\\
\mathcal{L}U_1(c) - \lambda[U_1(c) - U_0(c - P_s)] &= 0, \quad c \in (P_s, b_1^*), && \text{(E3)}\\
U_1(c) - U_1(b_1^*) + b_1^* - c &= 0, \quad c \geq b_1^*,, && \text{(E4)}
\end{aligned}$$

where again smooth pasting at $c = b_1^*$ is possible because the function defined by Equation (E3) has an inflection point at b_1^*. A particular solution to the non–homogenous Equation (E3) is

$$H(c) = \kappa_1 e^{\beta_1 c} + \kappa_2 e^{\beta_2 c}, \tag{E5}$$

where the choice of coefficients κ_1 and κ_2 allows for smooth pasting at $c = P_s$ and is such that the boundary conditions are satisfied. In particular, the mapping $c \mapsto H(c)$ also has a (unique) inflection point at $c = b_1^*$.

Observe that all solutions to the homogeneous equation $\mathcal{L}U_h - \lambda U_h = 0$ behave exactly as described in Section AppendixE.1, with $\rho + \lambda$ instead of ρ multiplying the zero–order term. In this case, however, the coefficients $A_{11}(b_1)$ and $A_{12}(b_1)$ used to define

$$U_1(c) = A_{11}(b_1)e^{\gamma_1 c} + A_{12}(b_1)e^{\gamma_2 c} + H(c) \tag{E6}$$

must be such that the homogeneous solution U_h satisfies $U_h'(b_1^*) = U_h''(b_1^*) = 0$.

The imposition of a regulatory lower bound on dividend distribution $b_{reg} > b_1^*$ results, when it comes to the corresponding particular solution H_{reg} to Equation (E3), in exactly the same behavior as we observed in Section AppendixE.1. Namely, b_1^* remains an inflection point and, since $H_{reg}'(b_{reg}) = 1$, then there exists $c_1 < b_1^*$ such that $H_{reg}'(c_1) = 1$. The corresponding homogeneous solution U_{h-reg} will also have an inflection point at $c = b_1^*$ and will, by an analogous argument, satisfy $U_{h-reg}'(c_1) = U_{h-reg}'(b_{reg}) = 0$. As a result, we conclude that imposing $U_{reg}'(b_{reg}) = 1$ at a point $b_{reg} > b_1^*$ results in a bifurcation of the solutions to the equation $U_{reg}' = 1$, with a second root at $c_1 < b_1^*$. Said differently, the equation $U_{reg}' = 1$ has, in general, two roots. The latter become a single, multiple one if and only if we choose $b_{reg} = b_1^*$.

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